UNIVERSITY OF PISA

SANT'ANNA SCHOOL OF ADVANCED STUDIES

DEPARTMENT OF ECONOMICS AND MANAGEMENT

Altruism and Endogenous Fertility

MASTER OF SCIENCE AND ECONOMICS

MASTER THESIS

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Acknowledgements

I would like to sincerely thank my Supervisor, Professor Luca Spataro, for the fundamental support he has given me for the analysis and complete reorganization of this Thesis. Many of the results I have achieved would have been impossible to be reached with my only efforts. I have accepted all Your suggestions, complaints and upbraids as a source for my personal and academic development. Hoping our collaboration could go further this Dissertation, I swear I will employ your advices for career, starting with my PhD.

I would also like to express my extreme gratitude to my Co-Supervisor, Professor Thomas Renström, for his useful insights and encouragement. The time you dedicated me during my Erasmus period at the University of Durham has been really appreciated. Moreover, I would also like to remark my complete beholden thought and beautiful memory to the entire staff of the University of Durham for the priceless support and understanding they have shown to me in any success and difficulty.

I am very fond to the Department of Economics and Management, the University of Pisa and Sant’Anna School of Advanced Studies for the chance they have given me to develop my critical thinking on a variety of economic issues and areas during my Master. My joining to These Institutions projects (Erasmus Project, Teaching Assistantship and Excellence Courses) has been fundamental for understanding how modern research is approached and taught to a class.

Finally, but of major importance, I’d like to thank my family and my friends (national and international) for the scientific and hearted support they have given me during all daily studies, achievements, difficulties and doubts this Thesis has brought me to face. This is our success.

Abstract

This paper contains an investigation of the modern literature on economic growth and endogenous fertility. The review of the most influential economic models is aimed at explaining the main forces that have been proposed as triggers for the so called demographic transition (that is the passage from a state of high fertility and economic growth to another one characterized by low fertility); children quality in the form of human capital, infant mortality and the introduction of social security systems will be fully analysed in this work. Moreover, a special attention is devoted one of the most famous model that is able to incorporate fertility choices into agents' behaviour, the Barro-Becker (1989) model. Finally, a data analysis will investigate the income-population relation, both on a Global and Regional perspective, in order to understand the inner causes of the demographic transition.

Altruism and Endogenous Fertility

Introduction

Some authors claim that the literature on population and economic growth is about as old as economic science itself (Ehrlich and Lui, 1997); fact the idea of measuring income in per capita terms and the analysis of economic consequences of population dynamics can be dated before the classics. However a group of studies concerning the relation between fertility and growth can be recognized after the seminal work by Malthus (1798), in which it is argued that the fertility rate is exogenously determined at the maximum natural rate and that population increases is detrimental for income per capita.

Nevertheless, this pessimistic conclusion is based to a number of crucial hypotheses (among all the exogeneity of crude birth and death rates) that denote what in literature is called the “Malthusian World”. The change of these assumptions has been the starting point for modern research on economic and demographic studies.

The availability of more reliable data on population and economic growth made scholars unveil the non univocal correlation between these two variables, and drove the literature to develop models in which fertility and economic quantities (e.g. income) are both endogenous.

Obviously, new theories had to get rid of some of the fundamental “classical” assumptions; the most relevant is the abandonment of land as a critical factor determining production constraints, and that is the reason why they are recognized as “neoclassical” growth theories.

The first attempts to reconcile growth with individual choices is due to the seminal works by Solow and Swan (1956), Koopmans (1965) and Cass (1965), according to whom the main driver is to be recognized in factors accumulation; however, most of these models keep on treating population growth and fertility choices as given and exogenous phenomena. These models predicts that population growth will not bring the economy to a subsistence level of income (like in Malthus). Instead agents' interest will ensure a rate of savings high enough to match (and overtake) the population effect, and lead the economy to an accumulation path (at least) sufficient to ensure per capita income growth (although this is not in line with what empirically observed by Coale and Hoover (1958) and Enke, 1960, 1976). Not surprisingly, the results of those theories is that the output level will be influenced by the exogenous socio-economic parameters, like the saving and the fertility rate (Solow and Swan, 1956), while its growth is entirely determinate by the technological progress. Consequently, it becomes self-evident that the so called "exogenous growth theories" cannot be effectively adopted for policy making.

Moreover, an interesting improvement of former theories of economic growth has been triggered by the OLG literature (see Allais, 1947 and Samuelson, 1958, further extended by Diamond, 1965). The basics of these theories lay in the analysis of an economy made by agents who live for a certain amount of periods and want to maximize a lifetime value function (in general an utility one), subject to an intertemporal budget constraint. The interesting point of these models is to be recognized in the heterogeneity of household characteristics over the lifecycle, generally expressed in the possibility of accumulating human capital and raising income in different periods of life. This premise is necessary to fully understand why this framework is particularly suitable for the developing of a theory of fertility choices.

In fact, the main scope of the introduction of fertility choices in growth models is to explain a widely accepted empirical regularity in many countries' fertility trends, the so called demographic transition. Its inner nature can be partially explained by the following sentence:

> *In traditional societies, fertility and mortality are high. In modern societies, fertility and mortality are low. In between, there is demographic transition. (Demeny, 1968, p. 520)*

According to this phenomenon, a certain force or group of forces triggered an incentive in the industrialized World to have less children in exchange for lower mortality levels (but the relation can be easily extended to many other socio-economic benefits, like higher income and human capital per capita or better welfare systems).

The first model to tackle the topic has been presented by Becker (1960).[1] In his work, the author endogenizes the choice on the number and the quality of children introducing this variable in the parents' utility function, together with income and parental knowledge on contraception; in this way, offspring is treated as a durable good. However, this framework presents some strengths and weaknesses; the main advantage is in it microfundation, but lots of inconveniences emerge because of the static structure of the model, according to which all decisions are taken in a single period of time.

Many other subsequent works, although inspired to the Becker (1960) assumptions, tried to overcome the shortcomings of the model. One of the most important is the Becker and Barro (1988) and Barro and Becker (1989), which will be presented in this thesis. More precisely, the work proceeds as follows.

In Chapter 1 we will analyse the different stream of thoughts upon the fertility choice theory with an altruistic parents set up; a special attention will be devoted to Becker-Barro reviews. In Chapter 2 we will present the Becker-Barro (1988) with its analytical setting and equilibrium properties; we will also focus on some comparative statics exercises of the model in relation to fundamental literature's results on altruistic parents and endogenous fertility. In Chapter 3 we will propose an empirical data analysis devoted to understanding the relationship between economic and demographic fundamentals, with a particular attention to the demographic transition process within several countries; different hypothesis on historical and contemporary data will be explained and tested on a qualitative and quantitative basis. Chapter 4 summarizes the results.

[1] Actually, others dealt with parental fertility choices before Becker; among all Banks (1954) and Liebesntein (1957). However, their works consider children as common consumer goods, instead of durables.

Chapter 1: Review of Altruistic Literature

In this Chapter we will deal with three different theories that try to explain the demographic transition phenomenon in frameworks with endogenous fertility and altruistic parents. First, we will be consider the effect of children quality and human capital on fertility; this approach leads to the well-known tradeoff between quantity and quality which will be developed in the first part of the Chapter. Second we will analyse the impact of infant and child mortality decline on the demand for children; this issue is particularly relevant when it comes to studying developing countries' economies characterized by high levels of infant mortality and fertility. Finally, the last part will analyse the effect of the introduction of social security systems on the demand for children. Although this aspect has received much attention in the case of selfish parents, it has some interesting implications in an altruistic setting, both on the side of responsiveness of fertility to changes in the economic context and sustainability of social security systems themselves.

1.1 Quality-Quantity tradeoff

One of the most common and surely most known relations that emerge from the demographic transition is the inverse relation between the number of children and various measures of their quality; in this sense, authors have used, both theoretically and empirically, different concepts to account for child quality, but mainly they can be grouped in two categories. The first one is the idea offered by Becker (1960), by which children quality has to be considered as the amount of resources spent in each child that contributes to parents utility; this literature will be briefly presented in the first part of this Chapter. The second and most important model is the one provided by Lucas (1988) and further developed by Tamura (1988) in an endogenous fertility setting, in presence of human capital; this concept is particularly useful for two reasons. First, it is more realistic and concrete, as it can be modelled to account for accumulation processes and externalities affecting it. The role of human capital in explaining the different development patterns of countries through time (between the Malthusian and the Modern Era) and space (between developing and developed countries) will be treated in the second part of this Chapter. Second it permits the introduction of some fundamental modern forms of education systems; a comparison between public and private provided schooling systems will be developed in the last part of this Chapter.

1.1.1 Children's quality

The seminal work on this field has to be attributed to Becker (1960). The concept of "quality" of children will play an important role for future developments, so that a clarification is required. Quoting Becker, "*I will call more expensive children "higher quality" children...To avoid any misunderstanding, let me hasten to add that "higher quality" does not mean morally better. If more is voluntarily spent on one child than on another, it is because the parents obtain additional utility from the additional expenditure and it is this additional utility which we call higher "quality."*"

Given this necessary premise, the result the author finds is that in modern societies the elasticity of quantity of children to income can be positive, but certainly lower than the value of the quality elasticity to income. This result has a huge impact on the responsiveness of fertility to economic growth, in such a way that the Malthusian monotone positive relation between income per capita and offspring quantity will be nullified. However, owing to its static characteristics, this model is capable of generating equilibria that reflect the emergence of the demographic transition, but certainly not its inner causes. Such a result can be explained in light of two main factors.

First, child mortality has fallen so low that, as parents are interested in the quality and quantity of surviving children, an increase in income will have little or no effect on cohort size. Second, improvements of economic condition experienced in last decades have been accompanied by an increasing awareness of contraception, leading to a reduction in unwanted procreations. Finally, as children characteristics are only those two previously recalled (quality and quantity), the effect of income that does not impact on quantity will surely influence positively quality.

The key features of this framework are also contained in Becker and Lewis (1973). In their model, parents are willing to maximize a generic utility function:

$$U(c, q, n) \tag{I.1}$$

Where c is the consumption of other commodities than children, q is the quality of each child and n is their number. The budget constraint is thus given by:

$$y = p_q nq + cp_c \tag{I.2}$$

Where y is the level of income, p_q is the unitary price of n children of quality q and p_c is the unitary price of consumption c. Results of this (Becker and Lewis, 1973) model are in line with Becker (1960). Moreover, this framework gives an explanation to the income elasticities described in Becker (1960); in fact, the shadow price of the quantity of children turns out to be increasing in their level of quality, and viceversa. Consequently, there is a quality-quantity tradeoff.

An interesting development of this theory is the work by Razin and Ben Zion (1975). In their work they investigate the effects of a public subsidy for child quality investments, finding similar results to those predicted by Becker and Lewis (1973); in fact, this increase in parental income will shift mostly to child endowment either than on their number. However, they also find that an increase in capital productivity will boost its accumulation and lower population growth rate. As a consequence, wages will rise and this has an uncertain effect on fertility. However, in order to achieve a proper microfunded intergenerational structure it is necessary to analyse the formulations by Abel (1985) and Becker-Barro (1986), which will be studied in Chapter 2.

1.1.2 Human Capital

The development of this literature takes place in models à la Becker-Barro with of pure parental altruism on one side, and the concept of human capital accumulation as an endogenous engine to growth (see Lucas, 1988). The seminal work to tackle this issue is the one by Becker, Murphy and Tamura (1990) (1994) and Tamura (1994), which reformulate what contained in Tamura (1988). The first two models (Becker, Murphy and Tamura, 1990; 1994) are based on the idea that parental resources (for instance, working time) are devoted to children human capital accumulation (what was first consider to be the quality), whose investments exhibit non-monotonically decreasing rate of return. As both child endowment and number are included as dynastic arguments of choice, the finding of the paper is in some sense in line with what found in previous seminal works; in fact it turns out that the basic relation between quality and quantity will continue to be negative due to the increasing returns hypothesis. However, the parental investment will affect also income per capita (as in Lucas, 1988), and this lead to the existence of three steady state equilibria. The first will be characterized by a stable Malthusian trap, in which households chose high fertility and low investments in children human capital; the necessary condition in order to make this equilibrium emerge is:

$$[a(n_u)]^{-1} > R_h \quad if \;\; H = 0 \tag{I.3}$$

Where H represents human capital per worker. In this case, the discount rate of future utilities $[a(n_u)]^{-1}$ is higher than the rate of return on human capital R_h. The second stable steady state is characterized by low fertility and abundant physical and human capita accumulation, and it is found when the following equation is satisfied:

$$[a(n^*)]^{-1} = R(H^*) \tag{I.4}$$

Which corresponds to the situation in which the discount rate of future utility $[a(n_u)]^{-1}$ is equal to the rate of return on human capital R_h. An unstable intermediate situation lays in between.

The second model (Tamura, 1994) proves the existence of an optimal value function when an endogenous discount rate of future utilities is taken into consideration. The results the author finds are consistent with what discovered by Becker, Murphy and Tamura (1990), but some additional features are strongly highlighted. First of all, the optimal value of consumption is positively related to the human capital investment, so that the stable equilibria exhibit an inverse relation between fertility and income growth rate; second, the parental level of human capital has a strong impact on their investments in children endowment (see also Becker and Tomes, 1986). This second result turns out to be in accordance with the empirical findings of many authors, among all Fernández and Rogerson (1996).

Tamura (1996) implemented the same model structure accounting for a non concave (as the works quoted before) and discontinuous value function, according to which the presence of a threshold h' in the level of human capital determines the shifting from a low accumulation pattern to another characterized by a higher level accumulation:

$$H_{t+1} = A(h_t + \gamma h^*)\tau_t \qquad if\ h_t < h' \tag{I.5}$$

$$H_{t+1} = A\overline{h}_t^{\delta}[(h_t + \gamma H^*)z_t]^{1-\delta} \quad if\ h_t \geq h' \tag{I.6}$$

Where $\overline{h_t}$ is the social human capital, $\delta \in (0,1)$ is the degree of its externality on the accumulation process h^* is the unskilled level of human capital and z_t is the amount of time invested in child. The dynastic parent at time t maximizes its utility function under the following constraint:

$$U(c_t, n_t, h_{t+1}) = \sum_{s=t}^{\infty} \alpha^{s-t}\{\alpha \ln(c_s) + (1-\alpha)\ln(n_s)\} \tag{I.7}$$

Where α is the parental degree of altruism. Moreover, the budget constraint can be expressed by:

$$c_t = [h_t + h^*][1 - n_t(\varphi + \tau_t)] \tag{I.8}$$

Where φ is the fixed time cost of raising children. The results of this paper are quite articulated and depict a quite reliable picture; the Becker, Murphy and Tamura (1990) (1994) equilibrium structure emerges, but some more implications are drawn on the side of income convergence and higher growth of less developed countries with respect to modern ones. Moreover, the empirical implications offered in the paper are very powerful mean in explaining the demographic transition phenomenon.

Nevertheless, those previous models rely strongly on the assumptions of imperfect capital markets; in such a situation the only way parents can invest in children human capital is via reductions of current consumption or fertility, because of the resources' constraint. Regarding this limitation, the same Becker and Tomes (1994) studied the implications of the introduction of an efficient capital system, so that even poor parents could borrow to invest in offspring quality and leave that debt as a negative inheritance (see also Barro, 1974). The model relies on the distinction between earnings deriving from human capital investments and from other sources of wealth (productive assets) left as a bequest, and on the assumption that the returns of the first are strictly higher than the second. In particular, earnings y_t deriving from human capital take the following form:

$$y_t = \theta(T_t f_t) h_t \alpha + \nu_t \tag{I.9}$$

Where θ is the return on human capital, T_t is the level of technological knowledge, f_t is the amount of human to non-human capital $\left(\frac{h_t}{k_t}\right)$, h_t is the level of human capital, αis the level of endowment common to all members of a given cohort and ν_t is the shock affecting earnings (luck in transmission of family endowments from a generation to the subsequent); it is assumed that y_t responds positively to T_t, but the contrary holds for f_t. [2] The second type of earnings is indirectly detected by its impact on the accumulation of human capital, so that a general form is attached to it:

$$H_t = \psi(k_t, e_{t-1}, s_{t-1}) \tag{I.10}$$

Where k_t is the total level of material inheritance, e_{t-1} and s_{t-1} are the parental and public expenditures in human capital respectively. As in previous works, the form of the inheritance transfer function shows how high endowed parents will have high endowed,

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[2] Note that this assumption on f_t is exactly the contrary of what imposed by Becker, Murphy and Tamura (1990) to achieve their results.

so that in principle the model would reproduce the same results of previous works. Hence, if the possibility for poor parents to borrow money from an efficient capital market is introduced, they find that the degree of intergenerational mobility in earnings would equal the degree of inheritability of endowments. However there are some problems related to this approach. First of all, poor families have generally little possibility to access credit for investing in offspring; moreover, the degree of intergenerational mobility in earnings depends on the number of children to divide the resources among, and this is particularly relevant when the first limitation holds.

A very long time prospective is taken into consideration by the unified framework by Galor and Moav (2002), who studied the evolutionary dynamics of a population with heterogeneous (determined by cultural lineage, for instance) preferences over the quality and quantity of children. The two main peculiarities of their model are the production function and preferences. On the technological side, Malthusian hypothesis are reproduced thanks to the introduction of a fixed factor X in production, which might be interpreted as land:

$$Y_t = H_t^{1-\alpha}(A_t X)^{\alpha} \tag{I.11}$$

Where Y_t is the total output, H_t is the level of efficiency units of labour and A_t is the level of technology.

The peculiarity of tastes relates to the possibility of having two different groups of individuals, a quality preferring and a quantity-preferring, distinguished by the weight β^i attached to child quality:

$$U(c_t, n_t, h_{t+1}) = (1-\gamma)\ln\left(c_t^i\right) + \gamma\left[\ln\left(n_t^i\right) + \beta^i \ln\left(h_{t+1}^i\right)\right] \tag{I.12}$$

$$0 < \gamma < 1$$

Where c_t^i is the household consumption of an individual of type i and generation t, n_t^i is the number of children of an individual in generation i and h_{t+1}^i is the level of human capital of each child.

Given this formulation, in the initial stage the quality-preferring group exhibits a higher fertility, rising the average human capital, because the resources constraint is burdening; this result is attained as the basic assumption is, as often happened in previous models, that children are time costly to be grown and educated. In the latter regime, the higher level of income per person will bring the quantity-preferring group to achieve a higher fertility at the expense of average quality, with a consequent reduction of human capital per offspring. This is a possible explanation to the different patterns of income-fertility characterizing the "Malthusian World" and the Modern Era.

On the side of the income-fertility relation, a very interesting work has been proposed by Moav (2005). This model considers a generalized Becker-Lewis framework, in which dynastic utility does not have a particular form and production functions of final output and human capital are not necessarily non-convex. In fact, the assumption of an increasing child rearing cost in their quality and of quality costs in the children number is maintained,[3] while it is assumed that the parents' productivity in educating offspring is increasing in their human capital stock. The economy is characterized by two-periods living individuals maximizing a utility function similar to that proposed by Galor and Moav (2002):[4]

$$u_t^i = (1-\gamma)\ln\left(c_t^i\right) + \gamma[\ln\left(n_t^i\right) + \theta\ln(wh_{t+1}^i)] \tag{I.13}$$

$$0 < \beta < 1$$

The accumulation function of human capital h_{t+1}^i is assumed to be an increasing and concave function of investments in children education in period t, e_{t+1}^i:

$$h_{t+1}^i = h(e_{t+1}^i) \tag{I.14}$$

According to all assumptions made so far, the result is that the relative price of quantity in terms of quality is an increasing function of parents' wage, which generates a comparative advantage for poor households to choose a large fertility, while driving

[3] This is what generates the non-convex budget set in the Becker-Lewis (1974) model.

[4] Note that the parameter β weights both the quality and quantity of children, but in particular it represents a relative weight in terms of consumption.

wealthy parents to invest in human capital. Finally, the non-convexity of the budget set works as an amplifying effect for advantages. Hence, results of the model are quite intuitive; the Becker, Murphy and Tamura (1990) prediction of the two stable equilibria framework still hold, though there are some inequality issues to be taken into account. While in the Becker et al. (1990) framework there is convergence among households in the same country, this model does not ensure this convergence. It might be the case that poverty is persistent also in a rich economy if income is unequally distributed. These results are in line with what empirically found by Perotti (1996), Barro (1999) and de la Croix-Doepke (2003).

Besides a number of empirical investigations on the quality-quantity tradeoff, some authors have discovered new peculiar features, emerging in some specific circumstances, according to which there might be a positive relation between quantity and quality of children. Yasuoka and Nakamura (2006) recognized such a situation from Japan experience as a consequence of some child care public supports. The basic framework is characterized by an economy populated by individuals maximizing the following utility function:[5]

$$U_t = \alpha \ln(n_t h_{t+1}) + (1-\alpha)\ln(c_{t+1}) \tag{I.15}$$

$$0 < \alpha < 1$$

Parents need a certain amount of time φ to rise their children, they spend x for any unit of education e_t and z for child-caring, hence the budget constraint will be given by:

$$x e_t n_t + z n_t + \frac{c_{t+1}}{1+r_{t+1}} = (1-\varphi n_t) w_t h_t \tag{I.16}$$

The human capital accumulation process h_{t+1} is assumed to lay both on the investment for education e_t and in the level of parental human capital h_t:

$$h_{t+1} = B e_t^{\varepsilon} h_t^{1-\varepsilon} \tag{I.17}$$

[5] It is self-evident that the altruism of parents is so pervasive that only the level of consumption of their children enter the utility function.

Then, the introduction of a child-care support policy is considered. This government measure will lower the child rearing cost, z, which will determine a fertility increase in the short run. However, the model shows how this policy is ineffective in the long run. Authors argue that the negative effect determined by the reduction in the relative cost of rearing children will increase fertility at the expense of its quality; as already explained by previous literature, this will tend to lower human capital accumulation in the long run, so that income per capita in equilibrium. As a result, the income effect will negatively impact on fertility, bringing it to the same level as if the child policy were not applied. On the other side, the effect of an educational subsidy will be in line with what foreseen by de la Croix and Doepke (2003) (2004).

Finally, a rather complete analysis of the problem is offered in the paper by Vogl (2013). Building the framework on Galor and Moav (2002), the model assumes a utility function in the following form:

$$U(c_t, n_{t,} h_{t+1}) = \beta \ln(c_t) + (1-\beta)[\ln(n_t) + \alpha \ln(h_{t+1})] \qquad \text{(I.18)}$$

$0 < \alpha < 1 \, , 0 < \beta < 1$

The human capital accumulation is a linear version of that proposes by de la Croix and Doepke (2003):

$$h_{t+1}(e_t) = \theta_0 + \theta_1 e_t \qquad \text{(I.19)}$$

so that the level of human capital is linear in the educational investment e_t during childhood. Finally, irrespective of human capital, each child costs τ units of parents time and k units of goods, so that the budget constraint is represented as follows:

$$c_t + kn_t + e_t n_t \leq wh_t(1 - \tau n_t) \qquad \text{(I.20)}$$

Where w is the wage in real terms and h_t is the parental human capital in efficiency unit. The model shows that the relation between skills and fertility is hump-shaped; as a consequence, given that earnings rise in the level of human capital, the positive impact of income on fertility is to be attributed to the fact that the largest share of the population laid in the segment where that function is actually increasing. Hence higher skills implied higher fertility. This would lead in the long run to an increase in the average level of human capital, to an increase in income and to a system in which parents have enough resources to invest in children education. This virtuous cycle brings the population to the part of the graph in which the relation between skills and fertility is actually negative, and this explains the common pattern experienced in the Modern Era. An empirical validation of the model is provided; the results are basically that the only significant component is just the level of skills, while other issues like social security systems and child mortality do not apparently play any role.

1.1.3 The effect of different schooling systems on human capital accumulation

The link between education and economic growth explained in previous models has been tested in a number of authors, among all Barro (1991), Barro and Sala-i-Martin (1992), Mankiw, Romer and Weil (1992) and O' Neill (1995); all of them find conditional convergence among a set of rich and fast developing countries in which education is a conditioning variable.

Thanks to these studies, the research interest focused on the main determinants for the accumulation of human capital and its policy implications, in particular on the side of the choice between a public or a private schooling system. Some earliest most remarkable contributions on this aspect are Glomm and Ravikumar (1992),[6] Eckstein and Zilcha (1994) and Zhang (1997); they analyse the effect induced by the introduction of some forms of public educational subsidies on human capital investments and fertility. Besides some differences in their framework, all of them find that the lower cost of education relative to consumption drives higher investments in human capital

[6] Although it considers fertility as an exogenous fixed parameter, it is worth quoting this work for the interesting idea of considering the impact of public and private schooling according to different distribution of income and human capital.

and higher growth, causing a negative impact on fertility rates. This seems to be in line with what predicted by previous models. However, if this policy is financed by the introduction of lamp-sum taxes on consumption or income, fertility will be increased as well; it is proven that its responsiveness to tax distortions is positive (Zhang, 1997).

Moreover, a similar effect will apply in retarding human capital accumulation, as a consequence of a reduction in its future rates of return (Eckstein and Zilcha, 1994). Eventually none of the effects is certain. With the purpose of investigating the net effect on both previous choices, Zhang and Casagrande (1998) develop an altruistic model that assesses both theoretically and empirically the results. Assuming a logarithmic utility of the following form:

$$V_t = \ln(c_t) + \rho \ln(n_t) + \alpha V_{t+1} \tag{I.21}$$

$$0 < \alpha < 1$$

Where V_{t+1} is the children welfare. The human capital accumulation process is assumed to be influenced both by parental human capital H_t and their private expenditure in children education e_t:

$$H_{t+1} = A e_t^{\delta} H_t^{1-\delta} \tag{I.22}$$

Each individual produces according to a linear production function $(1 - \nu n_t)H_t$ and allocates the output between consumption and investment in child education. The government imposes a flat consumption tax τ_C or an income tax τ_I in order to subsidize education at a flat rate s and finance its own not productive consumption; the authors assume that this consumption is fixed at γ_G in per old income terms. Hence the two budget constraints faced by agents and government are:

$$(1 - \tau_C)c_t = (1 - \nu n_t)H_t(1 - \tau_I) - (1 - s)e_t n_t \tag{I.23}$$

$$\tau_c \bar{c}_t + (1 - \nu \bar{n}_t)\bar{h}_t \tau_I = s \bar{e}_t \bar{n}_t + \gamma_G (1 - \nu \bar{n}_t)\bar{h}_t \tag{I.24}$$

Where $\overline{n}_t$, $\overline{h}_t$, $\overline{c}_t$ and $\overline{e}_t$ denote the average values of associated variables. The authors prove that the net impact on children education investments is positive, while the one on fertility is nil; the empirical testing validates these results.

An interesting departure from this framework is the model by de la Croix and Doepke (2004). Founded on the results contained in Mare (1997) and Fernández and Rogerson (2001), [7] and the framework by de la Croix and Doepke (2003), it considers the different impact on inequality and income growth due to different schooling systems, in the case in which parents choose fertility and human capital investments in their offspring, while fertility differentials are relevant. In their model they consider two-lived agents who care about their consumption c_t, the number of children n_t and their human capital h_{t+1}:

$$U(c_t, n_t, h_{t+1}) = \ln\left(c_t^i\right) + \alpha \ln\left(n_t^i h_{t+1}^i\right) \tag{I.25}$$

$$0 < \alpha < 1$$

Where α is the degree of parental altruism. Assume that child rearing absorbs a fixed quantity ν of parental working time and education is provided by teachers endowed with the same average human capital of the population $\overline{h_t}$ and financed by private parental investments e_t, so that the budget constraint is represented by:

$$c_t^i + n_t^i e_t^i \overline{h}_t^i = h_t^i(1 - \nu n_t^i) \tag{I.26}$$

The human capital accumulation process is the result of the effects implied by educational investments and parental human capital:

$$h_{t+1}^i = \mu\left(\theta + e_t^i\right)^{\eta}\left(h_t^i\right)^{\nu}\left(\overline{h}_t^i\right)^{1-\eta} \tag{I.27}$$

[7] The first one basically finds that fertility differentials per se are too small in the U.S. to have large effects on average education, while the second proves that the association of fertility differentials with the degree of marital sorting can lead to sizable long-run effects.

When a public education system is introduced, parents do not have to decide about private schooling investments e_t, but the government provides a common education service $\overline{e}_t^i$ for all the children by levying a proportional income tax τ_I. In the end, the new budget constraint results in the following:

$$c_t^i = (1-\tau_I)h_t^i(1-\nu n_t^i) \tag{I.28}$$

and the human capital accumulation is:

$$h_{t+1}^i = \mu\left(\theta + \overline{e}_t^i\right)^{\eta}\left(h_t^i\right)^{\nu}\left(\overline{h}_t^i\right)^{1-\eta} \tag{I.29}$$

These model's results are that public school tends to distort fertility to higher values keeping lower levels of education; this finding emerges as in this situation parents are not internalizing the cost for a higher number of children, and, in fact, income growth rate tends to be lower than in the case of private schooling. [8] Finally, another fundamental result is that, in case of household income heterogeneity, public school might be a preferable system to avoid high and low skilled fertility rate deriving from poor parents (this is the fertility differential), as this will negatively affect both income per capita and its distribution as well. For an important generalization of this model, see also Fan and Zhang (2013).

An interesting improvement of the stream of literature related with the model by de la Croix and Doepke (2004) has been proposed by Azarnet (2008). In particular, Azarnet criticizes the strong limitation of de la Croix and Doepke framework to the polar situations in which education is entirely provided either by a private sector or by the government. Instead, the author analyses the case in which parents can choose the optimal level of children's human capital h_{t+1}, the number of children n_t and

[8] The same result is attained by Palivos and Scotese (1996). They consider the implications of the financing of governmental services to children, such as education, when fertility decisions are endogenously determined. They show that, when the services to children are financed by taxation, the equilibrium outcome is upward biased from the socially preferred result toward higher fertility rates and lower economic growth, because each household internalizes the benefits, but not the costs of the tax-financed services.

consumption c, and than observe the impact on those choices of the introduction of a free educational system in various stage of development (hence now human capital accumulation is a general function of privately and publicly provided education). In particular, agents maximize a utility function of the form:

$$U(c_t, n_t, h_{t+1}) = (1-\alpha)\ln(c_t) + \alpha ln(wn_t h_{t+1}) \tag{I.30}$$

$$0 < \alpha < 1$$

Assume that parents incur a cost β in order to raise children and invest e_t in their private education, so that the budget constraint results in the following form:

$$c_t + w(\beta h_t + e_t)n_t \leq wh_t \tag{I.31}$$

As already said, the level of human capital owned by parents is crucial in many of the framework examined so far, and this leads basically to the two equilibria setting already discovered by Becker, Murphy and Tamura (1990). However, the introduction of a free schooling service may represent the only possibility for poor household offspring to accumulate enough human capital to escape poverty: in this sense the impact of such a system is proven to have positive effects on quality of children, as parents will not invest privately on their education because of lack of resources. On the other side, fertility choices will not be affected at all. If instead the household is enough well off to invest on children education privately, the effect of such a policy will be to displace investments from parents in favour of children quantity, but this effect has a negative impact on the aggregate level of human capital. A fundamental result of this model is that the quantity of human capital that each child gains from the public schooling system affects negatively that threshold under which parents do not add private educational investments to the public ones; this means that the efficiency of those public subsidies play a key role for the take-off from the Malthusian stagnant steady state.

2.2 Infant and Child Mortality

The importance of infant and child mortality has been recognized since the Malthusian model. In particular, the basic intuition behind this relation is to be searched in the desire of parents for surviving children, and not children in general. This idea, together with the consideration of the low mortality rates attained in recent times, have been developed qualitatively by Becker (1960) in order to prove that child mortality decline will have a sensitive effect in reducing the family size. However, its contribution is recognized through an indirect channel, according to which the reduction in mortality would reduce the cost of high quality offspring, lowering its number.

Given this premise, it is not surprising that for deeply understanding the main causes of demographic transition, a complete analytical study on the relation between child and infant mortality and fertility is required. In simple terms, two strategies to tackle this issue have been proposed. The first one is characterized by an exogenous treatment of mortality; agents face infant mortality as a parameter according to which they maximize their intertemporal choices. This situation is likely to happen in case parents believe that any measure undertaken on the side of children health care (either private or public) will be ineffective in increasing offspring survival probability. This setting will be developed in the first part of this Chapter. The second one assumes that parents perceive a possibility (strong or weak) for them to influence positively the survival probability of their children through investments in health care and sanitation (accompanied by other factors according to the framework, e.g. parental human capital). Besides being more realistic, this framework helps in explaining the emergence, even in developing countries, of various forms of publicly provided health systems and assistance programs for parents and children during the birth time. In this sense, as parents are concerned with the number (or the welfare) of surviving children, we can state that child and infant mortality is endogeneized in agents' choices.

1.2.1 Exogenous mortality

The first group of models dealing structurally with the mortality issue has been Ben-

Porath and Welch (1972), O'Hara (1975) and Ben-Porath (1976); while the first (Ben-Porath and Welch, 1972) considers differences between parental fertility choices under a certain and risky framework,[9] the others deal with changes in child and infant mortality and fertility responses in case of some expectations structures.

The model proposed by Ben-Porath and Welch (1972) is built on the assumption that lifetime is divided in 3 different periods and children have a probability p_1 of not surviving to the second period, p_2 to the third and p_3 of surviving for all the three periods. Than the representative household creating children maximize the following problem:[10]

$$E(U) = p_1 V(c, 0,0) + p_2 V(c, n, 0) + p_3 V(c, n, h) \tag{I.32}$$

$$s.t. f(n, h) + c = y \tag{I.33}$$

From the definition of the utility function given above, it is straightforward that a decrease in mortality leads to a relative increase in p_3 and an increase in the level of expected utility. As a result, the reduction in mortality will decrease the relative cost of quality in a greater proportion than the reduction of the price of quantity of children in terms of goods. The implication is that lower mortality shifts parents' choices to higher quality descendants. Moreover, in order to account for the demographic transition, O'Hara (1975) suggests that its explanation lies in the response of parents to mortality decline; precisely, the demographic transition's reestablishment of a low population growth rate requires that parents choose a smaller quantity of children after a mortality decline.

The second model considers, instead, a situation in which parents solve the following problem:

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[9] Actually the model does not make specific reference to the mortality issue, but it considers a model in which parents discriminate between male and female children in their utility function. The uncertainty is thus implied as far as the probability mass function of having a child of a certain gender is unknown, so that parents build expectations on it.

[10] Note that a fundamental assumption for all following results in the model is that the time distribution of Z and the household's wealth W are independent on how long children survive.

$$u[h(s,q)n,c] \tag{I.34}$$

Where s is the indicator of child survival probability distribution, e is the investment per child and q its price.[11] The budget constraint is expressed by:

$$qns + c = y \tag{I.35}$$

Where y is total family income. Moreover, it considers two different and, under some specific assumptions, complementary types of child mortality responses, the hoarding (also known as precautionary demand for children) and the replacement phenomenon; the difference between those two is that the first one is the adaptation of fertility choices to expected levels of mortality, while the second relates to the experienced levels. Using Israel micro data, the Ben-Porath discovers that the replacement effect can be predominant in case expectations on high child mortality is not widespread, so that parents respond mainly to actual and experienced offspring losses; on the other side, if this common though is shared, the hoarding effect will be higher.

Moving from the criticism to previous models,[12] Sah (1991) develops a model in which a discrete fertility choice is considered. In particular, children survive according to a binomial density of the form:

$$b(N,n,\pi) = \binom{n}{N}\pi^{N}(1-\pi)^{n-N} \tag{I.36}$$

Where N and n are the integer number of born and surviving children respectively, and $1-\pi$ is the mortality rate. The expected utility is derived from the number of surviving children, so that its functional form is given by:

[11] Note that just consumption of parents is considered, while that for child is neglected in favour of their number per quality factor.

[12] The critique develops in three ways. First expected utility depends on expected surviving children (see Pen-Porath, 1972, 1976), but this would lead to undesired results and fundamental contradiction with what foreseen by choices under uncertainty theory. Another important point is about the limiting specification imposed on the utility function, which is a quadratic one (see Newman, 1988). Finally the polarization of the analysis by O'Hara (1975), according to which the only interesting outcomes are those two in which all children die or all survive.

$$U(n, \pi) = \sum_N b(N, n, \pi) u(N) \tag{I.37}$$

Where $u(N)$ is the net utility from generating children. In fact it can be decomposed as a composite function of the difference between the parents benefit $g(N)$, as the happiness of giving birth, and the implied cost $h(N)$, say the expenditure for child bearing; for optimality purpose and for complying with previous literature, $g(N)$ is assumed to be increasing and concave, while $h(N)$ is increasing and convex. As a result, the optimality condition for n gives some important insights for the effect of a change in the mortality rate $(1 - \pi)$. In fact, just computing the cross derivative with respect to n and π it is possible to discover that the optimal fertility function $n(\pi$ is locally non increasing) is an increasing function of the mortality rate.

Another interesting paper that investigates the relationship between fertility and child mortality is Kalemli-Ozcan (2002). Starting from Sah (1991) survival function and Kalemli-Ozcan (2000), the author develops a quality-quantity tradeoff model in which mortality is uncertain;[13] in particular, it is assumed to have an OLG model where individuals live for two periods, whose expected utility function is given by:

$$E_t(U_t) = (1 - \alpha) \ln(C_t) + \alpha E_t\{\ln[n_t w h_{t+1}]\} \tag{I.38}$$
$$0 < \alpha < 1$$

The survival probability is modelled on the one offered by Sah (1991), so that it can be represented in its binomial form.

The results of this model fit quite well with the main features of demographic transition; precisely, an exogenous decrease in child mortality will induce a quality-quantity tradeoff in the economy. Owing to this, in developing countries where child mortality is very high, its decline will determine an increase in fertility. However, if returns on human capital are relevant and mortality is quite low (those are typical

[13] The author proves that the condition without uncertainty is pointless, as any exogenous decline in mortality will not affect either fertility of human capital investment (what the author calls the mean effect). However, the increase in the survival probability will certainly cause an increase in the population growth rate.

characteristics of developed countries), then the investment in child quality will become more attractive at the expense of fertility (and even the population growth rate may decrease if not becoming negative under some condition on education returns). Kalemli-Ozcan (2003) simply incorporated the same framework in a general equilibrium model.

An interesting extension of the model is offered by Fernàndez-Villaverde (2001). The peculiarity of this model is to fit qualitatively and quantitatively with the main features of demographic transition thanks to the compliance with the possibility of facing capital-specific technological change and capital-skill complementarity; in particular a continuous of household living for four periods is assumed, whose utility function is given by:

$$V_t(c_{t+1}^t, c_{t+2}^t, c_{t+3}^t, V_{t+1}) = u(c_{t+1}^t) + \beta E u(c_{t+2}^t) + \beta^2 E(c_{t+3}^t) + b(n_t) E V_{t+1} \qquad \text{(I.39)}$$

$$0 < b(n_t) < 1$$

Where c_{t+1}^t, c_{t+2}^t and c_{t+3}^t are consumption in the first, second and third period respectively, while V_{t+1} is the children future level of welfare and $b(n_t)$ the respective altruism factor toward children. About the budget constraint, it is assumed that individuals earn an unskilled wage w^u plus a skilled wage w^s for any unit of human capital endowment; moreover, they are allowed to purchase an a_j amount of non-contingent bond at time j, if they die before adulthood, their positive position is redistributed through a lump-sum tr_j to all the adults in the economy. Hence the constraints can be presented in the following form:

$$(1 + k_1 + k_2 s_t^t) c_{t+1}^t + a_{t+1}^t = (w_{t+1}^s h_t + w_{t+1}^u) l_{t+1}^t + tr_{t+1} \qquad \text{(I.40)}$$

$$c_{t+2}^t + a_{t+2}^t = w_{t+2}^s h_t + w_{t+2}^u + (1 + r_{t+2}) a_{t+1}^t + tr_{t+2} \qquad \text{(I.41)}$$

$$c_{t+3}^t = (1 + r_{t+3}) a_{t+2}^t + q_{t+3} \qquad \text{(I.42)}$$

$$l_{t+1}^t = (1 - e_{t+1} s_{1t} n_t) \qquad \text{(I.43)}$$

$$l_j^t \in [0,1]$$

Where r_j is the interest rate at time j and q_j is the capital tecnology factor. Using this framework, the author finds that the transmission mechanism proposed before does not fit with the fact of demographic transition; moreover an empirical test to support this hypothesis is provided.

A fundamental distinction between different types of young mortality is implicitly pointed out by Mateos-Planas (2002). In order to explain it, just assume an economy in which people lives for two periods and the survival probability between those two is represented by s; if a certain individual dies in the youth period (whose probability is obviously $1-s$), than there is a probability γ that he has generated children before dying. Finally assume that the expected utility function of each single household is given by:

$$V_t = E[c_t^{\sigma} + \beta n_t^{1-\varepsilon} V_t | t-1] \tag{I.44}$$

$$0 < \beta < 1 , 0 < \sigma < 1 , \varepsilon < 1$$

Where c_t V_t is the welfare of children; note that this form is nothing other than what offered by Barro and Becker (1989) under uncertainty assumption. The model focuses on the effect of longevity on the accumulation of physical capital, but it contains some interesting insights as far as the relationship between mortality and fertility is concerned; in particular, it is proven that a local increase in the parameter γ will tend to decrease the fertility rate n. Moreover, an increase in the survival probability will generally tend to increase the same parameter γ, so that, in the end, it is possible to state than a decrease in the mortality rate (especially at non infant ages, say near to the transition of an individual from the first to the second period) will tend to depress fertility.

In order to make the Becker-Barro model fit the facts, Doepke (2004) augments it with some realistic assumptions in order to study the effect of child mortality on fertility. Precisely, he studies three different formulations of the model: in the first one he presents it in the form proposed by Barro and Sala-i-Martin (2003), so that utility function is given by:

$$V_t(c_t, V_{t+1}) = \frac{c_t^{1-\sigma}}{1-\sigma} + \alpha(n_t)^{\varepsilon} V_{t+1} \tag{I.45}$$

$0 < \beta < 1$

Where c_t is α is the parental altruism and V_{t+1} is the level of children welfare. Given the absence of uncertainty, the number of surviving children $n = \pi N$, where b is the total number of children and s is the survival rate. The budget constraint is given by:

$$w \leq (p + qs)N + c_t \tag{I.46}$$

Where p is the cost of giving a child birth, q is the additional cost for the fraction of survived children and s is the deterministic probability for children to survive. The results associated to this specification are that if non-survived children are costless, then fertility is a decreasing function of the survival probability, while the contrary holds in the opposite case.

The second formulation takes into consideration the stochastic mortality issue, in the same way in which Sah (1991), Kalemli-Ozcan (2002) (2003), so that a binomial mortality rate is taken into consideration. The only result associated to this specification is just that, if non-survived children are costless, then fertility is non-increasing in the child survival probability.

Finally, the third model considers a sequential fertility framework (see also Sah (1991) and Wolpin, 1997),[14] which means that parents live for T periods and decide the number of birth according to the number of survived older children, provided they procreate before a certain period K (as they are not fecund afterwards). Assuming that a child that survives for the second period will surely reach adulthood and denoting by b_t the number of birth at period t, by y_t the number of young children (born in the previous period, so they can still experience child mortality) and n_t the number of older offspring. The recursive definition of the probability of having children in different time periods

[14] In Sah's model, costs accrue only to surviving children, there is no limit to fecundity, and children survive for sure once they make it through the first period. Wolpin (1997) analyses a three-period model (and employs a multi-period version for estimation) which allows for differential survival of infants and children and limits fecundity to the first two periods.

$(P_t(h_t))$ leads to the formulation of the following maximization problem:

$$\max_{\{b_t\}_{t=0}^{T}} \left[\sum_{t=0}^{T} \sum_{h_t \in H_t} \gamma^t \frac{(w - pb_t(h_t) - qy_t)^{1-\sigma}}{1-\sigma} P_t(h_t) + \beta \sum_{n=0}^{N} n^{\varepsilon} VP(n)\right] \quad (I.47)$$

Where $h_t = \{n_t, y_t\}$, H_t is a binary function that lets the individual chose whether to give a child birth or not at time t. The result associated to this formulation is that, if non-survived children are costless, then the number of children that will reach adulthood $(b_t(h_t)s_i)$ is non-increasing in their probability to survive for the first period (s_i, hence fertility in decreasing). Note that in both stochastic frameworks, fertility turns out to be a decreasing function of the survival rate.

Finally, the exogenous mortality framework is integrated with social norms concerning fertility by Bhattacharya and Chakraborty (2012); precisely, by social norms the authors mean that families will decide for their own fertility choices basing the process on a family-size ideal existing in the social environment in which they live. Aiming to effectively explain the empirical regularity according to which a reduction in child mortality is preceded by a fall in net and total fertility, the authors assume a utility function in the following form:

$$U(c, n) = (1 - \alpha)\ln(c) + \alpha \ln(\pi N) - \gamma\omega(d_n) \quad (I.48)$$

$$\gamma > 0\,, 0 < \alpha < 1$$

Where N is the total fertility rate, s is the survival probability, $d_n \equiv |n - n_s|$ is the "penalty function",[15] and n_s is the ideal family size. Children are supposed to bring some costs, β, only if they survive at birth, so that the budget constraint will be represented by:

$$c + \pi\beta N = y \quad (I.49)$$

Given this specification, it is proven that a reduction in child mortality will negatively

[15] It is called penalty function as ω is assumed to be an increasing function and the function d_n is an increasing function of the deviation from the social ideal n_s.

affect both total an net fertility rate, and this will work better as larger the family-size ideal is, and this can is acknowledged to be a very powerful child and population control policy.

Building on the idea proposed by Bhattacharya and Chakraborty (2012) of social norms, Canning, Gunther, Linnemayr and Bloom (2013) developed a model in which parents derive their utility from their own consumption c_t, survival and total children, $\pi_t n_t$ and n_t, and the relation with other families in the same group and $n_t - n_s$, their offspring human capital endowment h_{t+1}^C and their future wage w_{t+1}^C. The utility function is given by:

$$U(c_t, \pi_t n_t, h_{t+1}^C) = U_1(c_t) + U_2(n, h_{t+1}^C, w_{t+1}^C) + U_3(n_t, n_s) \quad \text{(I.50)}$$

as in Fioroni (2010), the human capital accumulation process is the result of a joint effect by parental human capital h_t^P and their investments in education e_t:

$$h_{t+1}^c = \varphi(e_t, h_t^P) \quad \text{(I.51)}$$

parents bear an additional cost for each child born equal to β, so that the budget constraint is given by:

$$c_t = m_t + w h_t^P (1 - (\beta + e_i) n_i) \quad \text{(I.52)}$$

Where m_t is considered to be a non-labour income. Finally, the child survival probability function is the same as the one proposed by Sah (1991) and Kalemli-Ozcan (2002) and Kalemli-Ozcan (2003). In the end, the response of fertility to changes in child survival mortality depends crucially on the elasticity; in particular, if the elasticity of fertility to child mortality is below the value of -[1], then there will be an increase in the total fertility rate, while the contrary will holds in the opposite situation.[16]

[16] It is really straight forward that this conclusion allows for different responses across countries at least.

1.2.2 Endogenous mortality

Notwithstanding a lot of strong empirical linkages between mortality and income per capita found by many works (see for example Yamada, 1985), all previous models deal with infant and child mortality as an exogenous variable, concentrating on comparative statics. On the other side, especially in the late 90's, many authors began to recognize the necessity to endogenize this aspect, owing to its relevant impact on fertility choices.

In this sense, Blackburn and Cipriani (1998) develop a model in which infinity-lived parents choose whether to invest their resources in the number of children n, in consumption c, or in health expenditure in order to increase survival probability of their offspring (this way mortality is endogeneized). The model is based on the BB framework, further implemented in continuous time by Barro and Sala-i-Martin (1995), so that the parents' utility function accounts for the family size N, the consumption c and the number of surviving children $(N - m)$:

$$U = \int_0^\infty e^{-\rho t}[\alpha \ln(N) + \log(c) + \phi \log(N - m)]dt \tag{I.53}$$

$$0 < \alpha < 1$$

Where N is the number of a typical dynasty, and $(N - m)$ is the number of surviving children. The peculiarity of this model is the possibility for parents to influence the mortality phenomenon, so that m is defined as:

$$m = M\mu\left(\frac{\varsigma}{k}, \frac{X}{K}\right) = M\mu(\varepsilon, \chi) \tag{I.54}$$

Where ς is the expenditure per child, k is stock of capital per person, X is the ratio of public health expenditure and K is the aggregate level of capital; the function governing the mortality rate is decreasing and convex in both argument. The optimization process brings to a negative relation between income per capita and fertility and income per capita and mortality, while it turns out to be positive between income and child expenditure (which in turn lowers mortality, ceteris paribus). As a result the specification of the model brings to a positive impact of child mortality on fertility behaviour.

A very interesting extension of the previous result is achieved by Cigno (1998). The model develops under two different hypothesis. The first is that parents believe that infant mortality is totally exogenous and they cannot do anything to reduce it; owing to its framework, the results are similar to those found in the works of Sah (1991) and Blackburn and Cipriani (1998), so that mortality and fertility go in the same direction. In the other situation, there is an assumption about the parents' belief to positively affect child probability to survive through, say, sanitation and nutrition. In light of this situation, Cigno endogenizes child mortality m and deals with a household maximizing the following expected utility:

$$E(U) = u(c) + \int_0^N f(n, N, \beta + \varsigma, s)\, dn = u(c) + g(N, \nu, s) \qquad \text{(I.55)}$$

Where β and ς are the time and health care cost incurred by parents to rise a child and $s = 1 - \pi$ is the survival rate, or better the mean of the survival probability distribution. Note first that the probability for children to survive is left to the realization of a certain random variable whose distribution is represented by the function $f(n, N, \beta + \varsigma, s)$. Second, and much more important, it is assumed that the perceived density of n is conditional on β, N and s.[17] According to the constrained maximization principle and the concavity of the value function it is possible to discover that the model can generate both positive and negative correlation between fertility and child mortality. In particular, when the probability for offspring to survive is high, then the gain derived from the increase in the marginal utility of an additional child the increase in that parameter s. This framework is capable of explain the sentence by Demeny (1968) and the concept of demographic transition. In developing countries infant mortality is generally very high, so that an exogenous reduction in the average survival probability drives a higher fertility and a higher investment in child education and health (ν), reinforcing the fall in m. While the country is developing, the negative effect of child mortality on fertility tends to vanish in favour of a positive one, which denotes the situation of developed countries. This result is generalized for any utility function form by Momota and Futagami (2000).

[17] Remember that in the case in which parents thing they cannot affect the mortality rate the parameter s=1-m is a constant.

In the same direction, Gómez (2001) analyses the effect of a health subsidy to reduce child mortality and one for the reduction of child rearing cost: dealing with infant mortality, this question comes out to be fundamental, especially when parental altruism is taken into account. In fact, the basic immediate impacts of those policies would be of increasing the total level of human capital endowment, which will positively affect the parents' utility function (that is the case of child rearing cost subsidy) or to increase the time span through which parents can derive utility by their children's welfare (that is the case of health subsidy).[18] Hence the convenience for policy-makers whether to provide one of the two forms of subsidies is not predetermined, as both seems to have a positive impact on societal welfare. In order to answer this question, the author develops the ideas formulated by Barro and Sala-i-Martin (1995) and Blackburn and Cipriani (1998), so that the household's utility function is expressed as:

$$U = \int_0^\infty e^{-\alpha t}\{\psi \ln(N) + \ln(c + \varphi g) + \phi \ln(n - d(\hat{g})\}\mathrm{dt} \tag{I.56}$$

Where g is the per capita health expenditure, and $d(\hat{g})$ is the family mortality rate; in particular, the mortality rate function $d(\hat{g})$ is assumed to be positive, monotonically decreasing and convex (diminishing returns to health expenditure).

Both health and child rearing policies can be financed through a capital income or consumption tax; the choice does not affect the impact of the introduction of child bearing subsidies (it tends to lower fertility in favour of higher human capital, see the previous paragraph on the quality-quantity tradeoff), but it is decisive in case of public health support. In fact, in case the policy is based on a consumption tax, the fertility rate will decline in the long run, while in case of a capital income tax, the fertility rate will exhibit a reversed hump-shaped behaviour, so that it will increase in the long run.[19]

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[18] This can have various forms, it can take into account the level of children's utility, or the level of human capital per child, or even the total level of human capital. All these specifications are attached with different degrees of altruism, from a pure one to and impure one and selfishness (when it is considered only parents' consumption).

[19] The net effect on population size will be positive in any case, as an increase in longevity will increase the cohort size, although offspring size may shrink.

An interesting analysis on the mortality factors is offered by Stulik (2004). In particular the author distinguished a completely external force driven by uncontrollable phenomena S, say the weather condition or geography,[20] and an endogenous one controlled through health expenditure. The utility function is expressed in a general form and depends on consumption in the two periods of individuals' life $\{c_1, c_2\}$, on the number of surviving children sN and on their quality h:

$$U = b_1 \ln(c_1) + b_2 \ln(c_2) + b_3 \ln(sN) + b_4 \ln(e + h) \tag{I.57}$$

$$b_3 > b_4$$

Where e is the investment in children education. Assuming z is the time cost of rearing children and given the two-period life, the budget constraint results in the following form:

$$c_1 = [1 - (z + \varsigma + h)N - \vartheta]w \tag{I.58}$$

$$c_2 = (1 + r)\vartheta w \tag{I.59}$$

ς is the part of income voluntary spend for children health, ϑ is the saving for old age consumption and r and w are the interest and the wage rate. The survival rate can be defined as a function of the expenditure in health through the following specification:

$$s = \min(1, S + (1 - S)\lambda\varsigma) \tag{I.60}$$

With $\lambda > 0$. Reasonably, there will be a positive relation between the survival rate and health expenditure, and in fact $\frac{\partial s}{\partial \varsigma} = (1 - S)\lambda > 0$. The model concludes that, in the weather favouring country, income needed in order to achieve an investment in children education will be much lower, because the survival rate will be enough low will a small fraction of resources spent in reducing child mortality. On the other side, in the worse

[20] The lower life expectancy at birth in tropical areas has been empirically proven by many authors, among all Bloom and Sachs (1998); the result is robust to income controls.

weather condition country, parents will dedicate the largest part, if not the totality, of their resources in reducing child mortality, as its high value and the assumption on b_3 and b_4 will permit the marginal benefit of investing in child health care to be strictly higher than the one deriving from investment in schooling.

An entirely different result is reached by Azarnet (2006). The main difference with the previous framework is to consider mortality as dependent on the relation between parental human capital and sanitation investments, and not on their health expenditure; moreover, infant death is considered (children die only before any educational process has been incurred), so that a decline in child mortality leads to a decrease in the level of education. In particular individuals derive utility from their consumption at adulthood c_t and from their children total income y_{t+1}:

$$U_t = (1-\alpha)\ln(c_t) + \alpha \ln(y_{t+1}^N) \tag{I.61}$$

$$0 < \alpha < 1$$

Adults are endowed with a human capital level h_t, and they can employ it for child rearing or working (gaining a wage w); offspring raising costs are distinguished in time and goods expenses for growing z_1 and cost related with surviving children education time z_2. Given that the number of births is n_t, the survival probability is π_t and the expenditure in children human capital can be expressed by e_t, the budget constraint will look like the following:

$$c_t + w[z_1 h_t + \pi_t(z_2 h_t + e_t)]n_t \leq w h_t \tag{I.62}$$

to complete the framework, the author considers a children income function of the following form:

$$y_{t+1}^N = \pi_t w h_{t+1} n_t \tag{I.63}$$

and a survival probability function:

$$\pi_t = \begin{cases} (\nu_1 h_t)^{\frac{1}{\alpha}} \ , \ if \ h_t < \frac{1}{\nu_1} \\ 1 \quad , \quad if \ \ h_t \geq \frac{1}{\nu_1} \end{cases} \tag{I.64}$$

the result of the model is that timing of child mortality is crucial for the determination of its impact on fertility. If infant mortality is taken into consideration, an exogenous decline will imply a reduction in fertility and education, but population will grow faster; however, this reduction in education will negatively affect human capital accumulation and long run growth, leading to the absorption of the exogenous fall in mortality. Finally, if parental investment in schooling became more productive, than human capital rising could eliminate child mortality in the long run.

Building on the basic framework by Stulik (2004) and the distinction between an exogenous and an endogenous mortality factor,[21] Stulik (2008) offers an explanation of the inverted u-shaped behaviour expressed by the delayed response of fertility to declines in mortality. This question is particularly important to understand the fundamental meaning of higher fertility rates in countries in which weather conditions lead to a very high child mortality. A standard two-period living individual is considered, and parents can partially affect mortality rates via child-care investments. First of all a subsistence consumption c_{sub} is considered, so that a level of consumption $c < c_{sub}$ basically leads to an infinitively negative utility; second, parents derive utility from the expenditure in children health ν:[22]

$$u(c, sn, d) = \log(c - c_{sub}) + \beta_1 \log\big(Ns(S, \varsigma)\big) + \beta_2 \log(\varsigma) \tag{I.65}$$

Where s is the survival rate composed by an observed (or extrinsic) part S and parental health expenditure for children ς. While the survival probability function draws directly from the basic framework, the budget constraint assumes a much simpler form:

[21] In the paper those two components are respectively called extrinsic and intrinsic; while the first one deals with the latitude of a certain Country, the second takes into consideration child health expenditure.

[22] This might be interpreted as a “joy of giving” phenomenon, although it is not a properly bequest.

$$y = c + N\varsigma y \tag{I.66}$$

Where y represents total income. Results are in line with what presented in the basic model; in general, there is a non-empty set of preferences according to which the worse condition country exhibits a higher fertility rate, and this situation is more likely to emerge as technological progress is lower and land is a more crucial factor of production.[23]

An interesting departure from what proposed in previous models is contained in Fioroni (2010), who studies the impact of child mortality reductions on fertility in different educational systems; starting from the idea of child mortality offered by Azarnet (2006) and a linear production function, the author considers individuals whose utility function is expressed in the following form:

$$U^i = (1-\alpha)\log\left(c_t^i\right) + \alpha\log(\, s_t^i N_t^i h_{t+1}^i) \tag{I.67}$$

$$0 < \alpha < 1$$

Where $s_t^i = s(h_t^i)$ is the survival probability function.[24]

Under a private education system, parents provide schooling for surviving children and the budget constraint will be given by:

$$c_t^i = h_t^i \left(1 - \frac{\nu N_t^i}{s(h_t^i)}\right) - e_t^i N_t^i \tag{I.68}$$

while the human capital accumulation process:

$$h_{t+1}^i = (\theta + e_t^i)^{\varepsilon}(h_t^i)^{1-\varepsilon} \tag{I.69}$$

Where e_t^i is nothing other than the education expenditure.

Instead, under a public education system, the government levies a proportional tax

[23] Note that these conclusions depict the so called "Malthusian World".

[24] As usual, $\frac{\partial \pi}{\partial h_t^i} > 0$.

on all adults and provides children a common schooling level $\overline{e}_t$, so that the budget constraint is:

$$c_t^i = (1-\tau_t)h_t^i\left(1-\frac{\nu N_t^i}{s(h_t^i)}\right) \tag{I.70}$$

however, we should even consider the Government budget constraint:

$$\overline{e}_t \sum_{i=1}^{P_t} n_t^i = \sum_{i=1}^{P_t} h_t^i \tau_t \left(1-\frac{\nu N_t^i}{s(h_t^i)}\right) \tag{I.71}$$

Where P_t is the total population at time t. The accumulation process for human capital is assumed in the following form:

$$h_{t+1}^i = (\theta+\overline{e}_t)^{\varepsilon}(h_t^i)^{1-\varepsilon} \tag{I.72}$$

As a result under a private education system, if income is low initially, the economy converges to a Malthusian stagnation steady state. For a high level of initial income, the economy reaches a growth path in which children's education rises and fertility decreases with income. In the growth regime under private education, exogenous shocks that lower child mortality are detrimental for growth: fertility increases and education declines. In contrast, under a public education system, the stagnation steady state does not exist, and health improvement shocks are no longer detrimental for growth.

1.3 Social security

Overlapping Generations Models are particularly suitable for analysing the effects of the introduction of social security and pensions systems; in fact, if we assume that one cohort of people is young and productive while another one is old cannot work, it becomes quite natural to think about the first one offering resources to the second one to finance its consumption. This particular social security setting is called unfunded or pay-as-you go system, as resources are levied, generally on some particular choices taken during adulthood (labour supply, cost of having children or consumption), right when they are spent for transfers. An alternative, although less common in reality, it to impose a labour tax on the same generation that receive resources back via pensions in the future, creating a transfer of resources under the form of constrained savings; this different setting is called funded social security system. The basic requirement for a social contract is respected in this Government managed plan as it can surely ensure the current working generation that the same treatment would be followed in the next period, so that they can take advantage of it as well.

As already specified in previous paragraph, there are several motives to have children, and, in general, some of these motives characterize specific development eras of the economy. While it is quite evident, from what written so far, that a form of private interest in investing in children's human capital to enhance their future welfare is more likely to be adopted in developed countries, some other forces may rule the choice of a developed and underdeveloped household. Hence, in the following paragraph we will deal with the effects induced by the introduction of funded and unfunded social security systems in an environment in which parents show altruism (according to various degrees) toward children. In the following part, we will treat works on the study and comparison between two different attitudes of parents in demanding children, altruism and selfishness.

1.3.1 Funded and Unfunded Systems

One of the first works to tackle the intergenerational resources transfer system and shifting of consumption from productive to non-productive periods is Razin and Ben-Zion (1975). Assuming an intergenerational utility function of the following additive and time separable form:

$$U_t = \sum_{t=0}^{\infty} \alpha^t U(c_t^1, c_{t+1}^2, n_t) \tag{I.73}$$

$0 < \alpha < 1$

Where c_t^1 and c_{t+1}^2 are the consumption during the middle-age and old age period respectively. The budget constraint will be given by:

$$n_t k_{t+1} = f\left[k_t - \left(c_t^1 - \frac{c_{t+1}^2}{n_{t-1}}\right)\right] \tag{I.74}$$

Note that in this simple case, the pension is assumed to be paid in kind, so that it is exactly equal to the old-age consumption, c_{t+1}^2. As a result, it can be shown that fertility depends positively on the amount of resources shifted to old generations.

Another interesting contribution is offered by Eckstein and Wolpin (1985), which compare the endogenously determined fertility level in a competitive economy to the fertility level that maximizes the steady state utility of a representative agent. Assuming that individuals derive utility from own consumption in middle and old age, c_t^1 and c_{t+1}^2, and from own children, n_t:

$$U = V(c_t^1, c_{t+1}^2, n_t) \tag{I.75}$$

They show that the optimal steady state utility (the one endogenously determined by individuals) yields a higher population growth rate when it converges to the Golden Rule allocation (the one determined by the social planner); this difference in fertility choices between the social planner and the decentralized economy case is an inefficiency. According to Samuelson (1958), this inefficiency can be absorbed by the introduction of a social security system based on the voluntary transfer of resource across generations; suppose, for instance, that the young generation gives up τ_t units of wealth in favour of the old generation, but it supposes to receive τ_{t+1} in the next period. Hence, assuming CRS production function $F(K_{t+1}, L_{t+1})$, imagine the Government implements a self-financed social security system in order to internalize the effect of individual decisions on fertility, so that the rate of return should be equal to the rate of fertility n_t; if this were the case, the budget constraint will be given by:

$$c_t^1 = w_t + K_{t+1} - \tau_t - \beta(1 + n_t) \tag{I.76}$$

$$c_{t+1}^2 = F(K_{t+1}, L_{t+1}) - w_{t+1}L_{t+1} - (1-\delta)K_{t+1} + (1+n_t)\tau_{t+1} \tag{I.77}$$

Where K_{t+1} is the physical capital saved for production in the next period, β is the cost of rising children, L_{t+1} is labour supply and δ is the depreciation rate of physical capital. An alternative policy would be to place a tax or subsidy on children financed by a lump-sum transfer between generations; however the optimal tax would need to be known by the government since such a program would not be voluntary.

Although the correlation between old-age provision and fertility is weak and often contradictory (Nelissen and van den Akker 1988; Swidler 1986; Nugent 1985), many other authors gave their contribution for understanding this relation in an altruistic model. For instance, Prinz (1990) proposed an OLG framework in which social securities are introduced in order to achieve the social optimum that characterizes a situation of mutual altruism. Hence assuming altruistic parents solving the following maximization problem:

$$\max_{(c_t^1, c_{t+1}^2, c_{t+1}^1, k_{t+1}, n_t)} U(c_t^1, c_{t+1}^2) + \alpha U(c_{t+1}^1, c_{t+2}^2) \tag{I.78}$$

$$s.t \quad c_t^1 \leq w(1-\tau_t) - \beta n_t - s_t - k_{t+1}(1+n_t) + k_t(1+r) \tag{I.79}$$

$$c_{t+1}^2 \leq w\tau_{t+1} + (1+r)s_t \tag{I.80}$$

$$c_{t+1}^1 \leq (1+n_t)[w(1-\tau_t) - \beta n_{t+1} - s_{t+1} - k_{t+2}(1+n_{t+1}) + k_{t+1}(1+r)] \tag{I.81}$$

$$c_t^1, c_{t+1}^2, c_{t+1}^1 \geq 0 \tag{I.82}$$

Where τ_t and τ_{t+1} are the social security tax and subsidy for young and old generations respectively, s is the total amount of savings while young and k is the level of bequest left from parents to children. Note that in this case we have three fundamental budget constraints as individuals will have to maximize their own consumption c_t^1 and c_{t+1}^2, and offsprings' consumption c_{t+1}^1. The result of the maximization process is a trade-off between the level of bequest k and fertility, n, so that the positive impact of the labour tax τ_t on the first will determine a decline on fertility; however, in this framework, it does not seem to be any effect by the magnitude of positive transfers to old ages τ_{t+1}.

An interesting contribution on the comparison between funded and unfunded social security systems is offered in Zhang (1995); in this paper, the author examines the effects of social security on the growth of per capita income in a model where investment in human capital of children is the engine of endogenous growth. Assuming a utility function of the following form:

$$V_t = U(c_t^1, c_{t+1}^2, n_t) + \alpha V_{t+1} \tag{I.83}$$
$$0 < \alpha < 1$$

Given bequests k_t and k_{t+1}, a child rearing cost β and a level of savings s_t, the budget constraint will be given by:[25]

$$c_t^1 = k_t + [1 - \beta(1 + n_t)]w(1 - s_t) \tag{I.84}$$

$$c_{t+1}^2 = (1 + r)s_t[1 - \beta(1 + n_t)]w - k_{t+1}n_t \tag{I.85}$$

The introduction of an unfunded social security system will determine a resource transfer τ_{t+1} of the following magnitude:

$$\tau_{t+1} = (1 - n_{t-1})\tau_t[1 - \beta(1 + n_t)]w \tag{I.86}$$

Where τ_t is the flat tax rate on labour income. Hence, the new system of budget constraints is given by:

$$c_t^1 = k_t + [1 - \beta(1 + n_t)]w(1 - \tau_t - s_t) \tag{I.87}$$

$$c_{t+1}^2 = (1 + r)s_t[1 - \beta(1 + n_t)]w + \tau_{t+1} - k_{t+1}n_t \tag{I.88}$$

[25] We've assumed the case of a small open economy, so that w and r are endogenously determined, just to simply the analysis in light our concern of focusing on impacts on fertility. This assumption is the same as the one used in Becker and Barro (1988).

As a result, unfunded social security may speed up per capita growth by reducing fertility and increasing the ratio of human capital investment per child to per family income when private intergenerational transfers are operative; this is due by the double effect of unfunded social securities on fertility. First, taxing labour income lowers the opportunity cost of spending time on rearing children by reducing the after-tax wage rates; second, transferring the tax revenue to the old increases bequests relative to income, and hence increases the cost of raising children, as pointed out by Becker and Barro (1988). Although private rates of return on human capital decrease because of the tax imposed on children future labour wage, the reduction of fertility is likely to offset this reduction in future income if α is enough high.

On the other side, the introduction of a funded social security system is mere delicate and demands for a further specification on the relation between generations. The first case to be analysed is that of assuming that receipts are linked to contribution, the new budget constraint will be given by:

$$c_t^1 = k_t + [1 - \beta(1 + n_t)]w(1 - \tau_t - s_t) \tag{I.89}$$

$$c_{t+1}^2 = (1 + r)(s_t + \tau_t)[1 - \beta(1 + n_t)]w - k_{t+1}n_t \tag{I.90}$$

In the other case, if we assume instead that receipts are not linked to contribution, the budget constraint setting will be equivalent to the Government budget will be given by:

$$\tau_{t+1} = \tau_t[1 - \beta(1 + n_t)]w(1 - r) \tag{I.91}$$

when voluntary saving is positive, a funded program is neutral in terms of fertility if receipts are linked to contributions, and most likely depresses per capita growth if receipts are independent of contributions. This difference is caused by the fact that when receipts are independent on contributions, agents are unable to realize the effects of their decisions on the return to the forced saving.

Zhang, Zhang and Lee (2001) has analysed the interaction of mortality decline, long-run growth and the provision of social security in an overlapping-generations endogenous growth model with actuarially fair capital markets. Assuming a utility function similar to that of Zhang and Casagrande (1998):

$$V_t = \ln(c_t^1) + \eta \ln(c_{t+1}^2) + \rho \ln(n_t) + \alpha V_{t+1} \quad \text{(I.92)}$$

$$0 < \alpha < 1$$

In period t, a middle-aged agent devotes βn_t units of time to rearing children, $\nu_t n_t$ units of time to educating children, and work for the remaining time l_t for a wage w. This agent receives a bequest with earned interest, $k_t(1 + r)$, from the old-aged parent at the beginning of period t, and leaves a bequest, k_{t+1}, to each child at the end of period t, so that children receive bequests regardless of their parents' survival status at old age. The middle-aged agent spends the earning and inheritance on own middle-age consumption c_t^1, saving via actuarially fair annuity markets $s_t l_t w$, so that budget constraints are given by:

$$c_t^1 = k_t(1 + r) + l_t w(1 - s_t) - k_{t+1} n_t \quad \text{(I.93)}$$

$$c_{t+1}^2 = (1 + r) s_t l_t w / \rho \quad \text{(I.94)}$$

Given this, the impact of social security systems is studied. Without social security or with fully funded social security, if parents value the number of children more than the welfare of children ($\rho > \alpha$), a rise in life expectancy reduces fertility (hence population aging comes both from longer life and from lower fertility), stimulates human capital investment and per capita growth. If parents value children welfare more than the number of children, a rise in life expectancy raises fertility (and hence has an ambiguous effect on population aging), reduces human capital investment and slows down per capita growth. If parents value the number of children as strongly as their welfare, a rise in life expectancy has no effect at all on fertility, investments and growth, even though saving and bequests may account for different proportions of physical capital investment. When unfunded social security is considered, a rise in life expectancy raises the social security contribution rate and has in general ambiguous effects on fertility, human capital investment and growth. An interesting result in this case is that a rise in life expectancy tends to have a negative effect on fertility and positive effects on human capital investment, growth, and the social security contribution rate for a wide range of plausible parameter values.

An interesting development of Zhang and Zhang (1995) framework is offered by Yakita (2001). In particular, the author focuses on efficient capital and private annuities markets in the presence of lifetime uncertainty; assuming a weak altruism of parents toward their children à la Eckstein and Wolpin (1985):

$$U = \log(c_t^1) + \alpha \log(1 + n_t) + \frac{1-\lambda}{1-p} \log(c_{t+1}^2) \tag{I.95}$$

Where λ is the probability for adults to die before retirement. The economy is composed by one-sector endogenous growth model which includes externalities from physical capital. The main conclusion is that an increase in life expectancy lowers the fertility rate and increases savings for consumption during retirement. In a growing economy, social security financed on a pay-as-you-go basis seems to raise the fertility rate and decrease life-cycle savings when elderly parents do not get support from their children, since it redistributes income from working generations to retired generations. However, it is shown that any social security system, regardless of its contribution rate, will not reverse the effects of an increase in life expectancy on the fertility rate and on per-capita growth which would occur even in its absence.

On the side of social security systems programming, Groezen, Leers and Meijdam (2003) proposed a framework in order to claim that pay-as-you-go schemes and child allowance policies are “Siamese Twins”. Starting from the work by Eckstein and Wolpin (1985), they analyse the optimal fertility choice of an individual maximizing the following utility function:

$$U = \log(c_t^1) + \alpha \log(n_t) + \theta \log(c_{t+1}^2) \tag{I.96}$$

Assuming that the Government is not present in the economy and denoting as β and s_t the child rearing cost and the level of saving by young generations, the individuals’ budget constraint can be represented by:

$$c_t^1 + \beta n_t = w - s_t \tag{I.97}$$

$$c_{t+1}^{2} = (1+r)s_t \tag{I.98}$$

The authors show that this maximization process will lead to a Pareto efficient allocation, regardless of what shown by Cigno (1993).

Instead, if a social security system is engineered by the Government, it should be accompanied by a child allowance program in order to achieve the same First-Best solution. Hence, assuming a pension system that transfers τ_t resources from the young generation to the old one through τ_{t+1} pensions; however, if this were the case, the economy will suffer the situation outlined by Cigno (1993), so that a child allowance program is needed. This policy is simply implemented by a subsidy φ aimed at reducing the cost of rising children β financed by a lamp sum tax ϑ_t. Hence the new budget constraint

$$c_t^{1} + (\beta - \varphi)n_t = w - \tau_t - \vartheta_t - s_t \tag{I.99}$$

$$c_{t+1}^{2} = (1+r)s_t + \tau_{t+1} \tag{I.100}$$

Hence, individuals will make the same fertility choice as the one characterized by the absence of the Government. This result is attained as the two externalities delineated by Cigno (1993) will perfectly cancel out each other. In fact, the benefit of an additional child for future output will be completely offset by the reduction of the capital–labour ratio (or per capita debt in a small open economy), so that parents who do not take these externalities into account are not likely to give birth to a (socially) suboptimal number of children.

An interesting reformulation of child allowances policies proposed by Groezen, Leers and Meijdam (2003) and uncertainty on the lifetime side is contained in Mochida (2005); in their model, the endogenous engine for growth is considered to be children human capital h_{t+1}, influenced by parental investment e_t in schooling and their human capital h_t á la Zhang and Casagrande (1998). Given this, altruistic parents will maximize the following utility funcation:

$$u_t = \gamma \ln(c_t^{1}) + \delta\lambda \ln(c_{t+1}^{2}) + \alpha \ln(n_t h_{t+1}) \tag{I.101}$$

Where λ is the survival probability coefficient.
In the absence of bequest motives, individuals are willing to invest their assets in such insurance, so that insurance companies promise individuals a payment of $\frac{(1+r)}{\lambda}a_t$, where r is the interest rate, is in exchange for having an estate a_t.
Thanks to the Pay-as-you-go system, individuals will pay a tax rate τ_1 on working time and get a pension τ_2. Denoting by l_t and w the labour supply and its remuneration, by β and φ the (time) cost of rising children and the child allowance policy, the budget constraint will take the form of:

$$c_t^1 = (1-\tau_1)wl_th_t - (\beta-\varphi)n_t - a_t \qquad \text{(I.102)}$$
$$c_{t+1}^2 = \frac{(1+r)}{\lambda}a_t + \tau_2 \qquad \text{(I.103)}$$

As a result, the introduction of a child-allowance policy and a PAYG pension system increases the number of children and decreases the labour time; moreover, when child allowances are given to parents depending on the number of children, parents have an incentive to increase the "quantity" of children without maintaining the "quality" of each child.
Instead when the government introduces the PAYG-pension system only, the introduction has no effects on the education time per child and the per-capita growth rate.

In line with what Groezen, Leers and Meijdam (2003) discovered, Gavhary (2009) found that fertility externalities can be internalized through a child allowance (or tax) or a linkage between pension benefits and the number of children. In fact, it is often argued that in the presence of a pay-as-you-go (PAYGO) social security system, there is a positive externality associated with having children which, if not corrected, implies that the number of children in a decentralized economy would be suboptimal. At the same time, it is also argued that a declining fertility will shrink the tax base and undermine the financial solvency of the PAYGO pension systems. To counter this, many economists have recently advocated a policy of linking pension benefits, and/or contributions, to the number of children (see Bental, 1989 and Fenge and Meier, 2005).

The peculiarity of this model is that of introducing parents' heterogeneity in the cost of rising children; hence two groups of individuals will be considered and they will be denoted by the subscript j.

Hence just assume that parental middle and old-age consumption, $c_{1,j}$ and $c_{2,j}$, and the number of their children, n, enter parents' utility function in an additive and separable form. Now introducing parents heterogeneity, we can consider two categories, so that β_j and y_j denote the cost of rising children and the steady state value of income for them. Finally suppose, instead, that the Government introduces a PAYG system in which $\tau_{1,j}$ resources from the young cohort in favour of the old one as a pension $\tau_{2,j}$ and a child allowance φ_j, so that the parental budget constraint can be represented in a compact form:[26]

$$y_j = \tau_{1,j} - c_{1,j} + \frac{c_{2,j} - \tau_{2,j}}{1+r} + (\beta_j - \varphi_j)n_j \qquad \text{(I.104)}$$

Then, as in Groezen, Leers and Meijdam (2003), this framework will lead to a Pareto efficient allocation. However, this prescription rest crucially on the assumption that no parents are better than others in raising their children and that fertility can be perfectly controlled. When either of these two assumptions are violated, the case for such policy recommendations are greatly weakened.

An interesting development on the side of child care and social security sustainability policies is offered by Hirazawa and Yakita (2009). In their model, the authors assume that children can be raised through parental time, z_t, or market provided services (like nurseries or baby-sitting), x_t, so that we can define a fertility function n_t:

$$n_t = \omega(z_t)^{\gamma}(x_t)^{1-\gamma} \qquad \text{(I.105)}$$

Given this, parents make their choices maximizing a utility function:

$$U = \log(c_t^1) + \alpha\log(n_t) + \theta\log(c_{t+1}^2) \qquad \text{(I.106)}$$

[26] Note that this setting requires the Government to recognize the two categories of parents in order to have optimality.

$\alpha, \theta \in (0,1)$

The existence of a PAYG system, by which τ_t^1 resources are transferred from the young cohort in favour of the old one as a pension τ_{t+1}^2, gives a system of budget constraint of the following form:

$$c_t^1 + s_t + x_t = (1 - \tau_t^1)w(1 - z_t) \tag{I.107}$$

$$c_{t+1}^2 = rs_t + \tau_{t+1}^2 \tag{I.108}$$

Where s_t is the amount of savings gathered during working time period.

In the standard case of children raised only with parents' working time, an increase in the payroll tax rate τ_t^1 means a decline in the after-tax wage rate and thereby induces individuals to increase parental child-rearing time z_t, thus reducing the labour supply. However, when individuals can substitute parental child care with market child care x_t, the increased parental child-rearing time does not necessarily result in an increase in the number of children (see Apps and Rees, 2004).

In fact, the effect on fertility will depend on the joint effect of three factors. The first one is the standard intergenerational redistribution effect from the working generation to the retired generation through the social security system (depends on the difference between the interest rate and fertility). Then, the (implicit) subsidy effect through tax-exemption of child rearing at home (tends to reduce children demand as consumption goods, as $(1 - \tau_t^1)w(1 - z_t)$ is reduced). Finally, the price effect through changes in the relative price of market child care (raises the fertility rate through declines in the opportunity cost of child rearing when market child care is produced by using market goods). Thus, when the population growth rate is sufficiently high, the intergenerational redistribution effect is great enough to dominate the subsidy effect and increase full income. The positive income effect together with the price effect raises the fertility rate.

At the same time, changes in the labour supply and fertility affect intergenerational income distribution through the social security scheme. Although increased parental child-rearing time tends to reduce tax revenue and hence social security benefits, the increased tax rate raises the fertility rate if the standard effect of social security on the steady-state income is sufficiently great. The reduced after-tax wage rate also decreases the cost of children by making the price of market child care higher relative to the cost of parental child rearing, thereby exerting a positive effect on fertility.

Finally, a complete work on the relation between social security systems and fertility is offered by Ling Yew and Zhang (2013); in their model, the authors use social security and education subsidization to eliminate the efficiency losses of human capital externality for the social optimum in an endogenous growth model with fertility, life-cycle saving and human capital investment. Assuming altruistic parents caring about middle and old-age consumption, c_t^1 and c_{t+1}^2, and number of children, n_t:

$$V_0 = \sum_{t=0}^{\infty} \alpha^t U(c_t^1, c_{t+1}^2, n_t) \tag{I.109}$$

$$0 < \alpha < 1$$

Suppose that parents have a unit of time to allocate between child rearing and working for a wage w; the cost, in terms of time, for each child is z, so that the fertility upper bound $N = \frac{1}{z}$ and labour income is denoted by $(1 - \tau_t^1)h_t w(1 - zN)$, where τ_t^1 is the social security financing flat tax and h_t is the level of parental human capital. Each parent will allocate labour income and bequest k_t into consumption, savings for old age s_t and children education $(1 - v_t)e_t$, where v_t is the subsidization rate. Finally, an old parents will finance consumption with their own savings (capitalised at an interest rate r), pension τ_{t+1}^2 and leave an even bequest k_{t+1} to all children. Hence the system of budget constraint is given by:

$$c_t^1 = k_t + (1 - \tau_t^1)h_t w(1 - zN) - s_t - (1 - v_t)e_t n_t \tag{I.110}$$

$$c_{t+1}^2 = s_t(1 + r) + \tau_{t+1}^2 - k_{t+1} n_t \tag{I.111}$$

On the side of human capital accumulation, authors assumed that it is the result of parents' level of human capital h_t, education investments and the externality given by the economy-wide average level of human capital, $\overline{h}_t$:

$$h_{t+1} = h(h_t, e_t, \overline{h}_t) \tag{I.112}$$

As a result, education subsidization for a lower education cost appears to be an ideal means to tackle the under-investment and the over-reproduction caused by the externality. However, the accompanying tax on wage income counteracts the positive effect of education subsidization on human capital investment and the negative effect on fertility.

The net effects of social security on fertility and on human capital investment depend on preferences. On the one hand, social security raises the time cost of children through the foregone earnings-dependent social security benefits, and raises the bequest cost to ease the increased tax burdens on children. On the other hand, the labour income tax for social security reduces both the time cost of a child and the return on human capital investment.

If the taste for the welfare of children is sufficiently strong relative to the taste for the number of children, then the net effect of social security on fertility is negative, in line with the empirical evidence in Zhang and Zhang (2004).

1.3.2 Altruism VS Selfishness

One of the pioneering works in dealing with selfish parents and pension systems is Leibenstein (1957). According to the "social security hypothesis" formulated in this work, the reasons for parents to demand children is that of supporting their old-age consumption. In fact, this system would work as a resources transfer mechanism for periods in which the agent is unproductive and have to rely on working-age savings, perhaps hard to be gathered because of stringent budget constraints, given of absence of capital markets. This literature has received many contributions (see Nishimura and Zhang, 1992, Wigger, 1999 and Yoon and Talmain, 2001); its predictability power in analysing developing countries actual and expected economic fundamentals (just remember that most of these economies strongly rely on the availability of abundant and cheap labour force, s.c Demographic Window) made this approach very attractive. The most common instrument to introduce such a ruling mechanism in family choices is that of assuming a reverse (from children toward parents) of a two-sided (from parents toward children and vice versa) altruism.

However, the same power in explaining fertility choices has been found even in case of altruistic parents. According to Cigno (1992), also altruistic parents may take advantage of resource transfers from their children without becoming selfish, but this should be supported by some forms of social contract that guarantees that the transfer of resources will be maintained in subsequent periods. Moreover, this altruistic setting is supported in some of its main conclusions. First of all the positive dependence of fertility on the interest rate; while the non-altruistic side of the theory states that an increase in the interest rate will determine a relative increase of young-age consumption (because of an increase in the subjective discount rate of future utilities) and a decrease in fertility, the altruistic formulation foresees the opposite. That is due to the positive cross-substitution effect and the positive income effect (but this actually depends on the specific formulation of the model) generated both on the size and the per-capita consumption of offspring.

One of the most complete studies on the relation between social security and fertility is offered by Cigno and Rosati (1992). In particular, they study the effect of existing fully-funded social security systems and their changes in coverage on fertility choices and savings according to different motives for parents to reproduce. Hence, the first hypothesis is of having selfish individuals that plan their resources allocation by maximizing the utility derived by their own consumption among childhood, adulthood and seniority, c_t^1, c_t^2 and c_t^3.[27] Moreover, they have full access to capital markets in order to secure their old age consumption (for which they do not perceive labour income) and give loans to children.[28] Hence, denoting by y_t and d_t the transfers that an

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27 Note that in this model it is introduced a "youth" consumption c_t^1, although there is no income. This can be interpreted as the loan in kind that an individual receives from parents

28 This point is fundamental. When pure selfish motive rules fertility choices, bequests are replaced by loans that agents can give to children in order to increase their consumption possibilities.

individual receives during adulthood (labour income) and old age (repayment of the loan given to children for consumption c_{t+1}^1), the budget constraint can be expressed as:[29]

$$c_t^2 + d_{t-1} - n_t c_{t+1}^1 = y_t \tag{I.113}$$

$$c_t^3 = n_t d_t \tag{I.114}$$

However, individuals may have an incentive to renege on their debt to parents if there were a capital market that allowed him to provide for his own old age by buying assets; such a behaviour will lead to the exclusion from family's transfers.

On the other side, it may be assumed that parents are altruistic toward children, so that their utility function will take into account the fertility rate n_t and children utility U_{t+1}:

$$U_t = U(c_t^1, c_t^2, c_t^3, n_t, U_{t+1}) \tag{I.115}$$

Or, according to Wildasin (1990):[30]

$$U_1 = U(c_1, c_2, n) \tag{I.116}$$

Hence, as loans are no more considered, we denote k_2 as the bequest left to children and τ_i the net transfers from the Government to generation i (difference between social security contribution and pensions), so that the budget constraint for the two generations are:

$$c_1 = y_1 + \tau_1 - \frac{k_2}{1+r} \tag{I.117}$$

$$c_2 = y_2 + \tau_2 + \frac{k_2}{n} \tag{I.118}$$

[29] Obviously, although consumption during youth is considered and enters the utility function, there is no budget constraint for it as it is an object of choice for parents born at time $t-1$ through transfers.

[30] Fundamental results can be derived in this simplified utility version, where only two generations are considered and lifetime periods are reduced to one.

Moreover, the Government should keep transfers in balance, so that:

$$\tau_1 + \frac{n}{r}\tau_2 = constant \tag{I.119}$$

Saving and fertility equations are then estimated from Italian time series data, using as explanatory variables the market rate of interest, the social security deficit, various measures of capital market accessibility and social security coverage, and a number of income and wage variables. Particularly worthy of note is the result that a fully-funded increase in social security coverage raises saving, while an increase in the social security deficit has the opposite effect. The empirical findings appear to support the assumption that fertility is endogenous and jointly determined with saving, and to favour the hypothesis that individual decisions are motivated by self-interest rather than intergenerational altruism.

On the same line of Cigno and Rosati (1992), Zhang and Zhang (1998) studied the effect of the introduction of an unfunded social security system when parents have different motives for having children. Apart from the non-altruistic case, the author analyses two fundamental cases of altruism, a weak and a strong one. In the first (altruistic) case, parents care only about the number of children, so that the utility function has the same expression as the one by Eckstein and Wolpin (1985):

$$U = V(c_t^1, c_{t+1}^2, n_t) \tag{I.120}$$

In the second (selfish) case, parents care not only to their number of children, but even to their future of welfare, as in Razin and Ben-Zion (1975), Cigno and Rosati (1992), Caballe (1995) and Zhang (1995):[31]

$$V_t = U(c_t^1, c_{t+1}^2, n_t) + \alpha V_{t+1} \tag{I.121}$$

$$0 < \alpha < 1$$

[31] Although the utility form is the same, Caballe (1995) did not take fertility as an object of choice for parents.

Given a child rearing cost $\beta(1+n_t)^{\varepsilon}$ with $\varepsilon > 1$ and a level of savings s_t, the budget constraint will be given by:

$$c_t^1 = (1-\delta-\tau_1)[1-\beta(1+n_t)^{\varepsilon}]w - s_t \tag{I.122}$$

$$c_{t+1}^2 = (1+r)s_t + (1+n_t)\delta[1-\beta(1+n_t)^{\varepsilon}]w + T_{t+1} \tag{I.123}$$

Where δ is the share of labour income voluntary given from children to parents and T_{t+1} is the pension transfer:

$$T_{t+1} = (1+n_t)\tau[1-\beta(1+n_t)^{\varepsilon}]w \tag{I.124}$$

As a result when intergenerational transfers are positive, social security reduces fertility but savings may not be affected; it is not surprising that social security increases per capita income growth. When intergenerational transfers are zero, social security reduces both fertility and savings, so that the effect on income per capita growth cannot be determined univocally. However, intergenerational transfers will be affected by the labour tax imposed to finance the social security system. In fact, if children are treated as pure capital goods (the case of strong altruism), gifts from children to parents must be positive no matter how high the social security tax becomes. If children are consumption goods as well as capital goods (the case of weak altruism), however, gifts may become zero when social security tax is sufficiently high and may even become negative if the social security tax increases further.

Chapter 2: The Barro-Becker Model (1989)

2.1 Introduction

The Barro-Becker (1989) model analyses fertility choices of altruistic individuals in a closed economy, so that the wage and interest rate are determined by the level of capital accumulated in the system. However, because of the Representative Household hypothesis, agents will take these parameters as given for any single period (perfect information on their time path) during the maximization problem. The altruism hypothesis permits agents to be considered as infinitively lived, though the model configures as an OLG in which individuals live for just two periods. The optimization process implies a non-arbitrage condition for consumption across generations, while fertility decisions are governed by the equality between the marginal benefit of an additional child and its rearing cost. As a result consumption growth depends on changes in the child-rearing cost, but not on the interest rates or time preference. Fertility choices instead depend positively on the World's long-term real interest rate and the degree of altruism.

The Chapter develops as follows. First we will set up the constrained utility maximization under a dynastic budget constraint. Second we will consider the effects of child mortality on fertility choices and consumption pats across generations. Finally we analyse the equilibrium of the model.

2.2 The model

In the first paragraph we will set up the individual problem for optimally choosing consumption and fertility in any period; this maximization process will be assimilated to the choice of an individual born at time $i = 0$ that determines the optimal allocation of resources (initial wealth and labour wage) over an infinite time horizon thanks to the assumption of altruism toward children. The framework is more general with respect to what presented in Becker and Barro (1988) in the assumption of a closed economy where the wage and the interest rate is determined by competitive markets.

In the second paragraph we will develop some comparative static analyses for studying the effect of a change in exogenous variables (like the initial wealth or the cost of rising children) on consumption and fertility choices.

2.2.1 The maximization process

Assume each individual lives for two periods; in the first period, during childhood, it is supported by parents, while in the second one, during adulthood, it supplies labour and gives birth to the next generation with no need for marriage. An agent born at time t derives utility U_t by its own consumption c_t, the number of children n_t and their utility $U_{i,t+1}$, $i = 1, \_, n_t$. Assuming additivity and separability of the utility function, we could give the following general specification:

$$U_0 = v(c_0, n_0) + \sum_{i=1}^{n_t} \psi\left(U_{i,1}, n_0\right) \tag{II.1}$$

Where $v(c_0, n_0)$ is the standard instant utility ($v_c > 0$ and $v_{ii} < 0$, $i = c_0, n_0$) and $\psi > 0$ is the altruistic function.[32] Since children concur equally to the maximization of parent's utility, we can state that the optimal choice will confer the same utility level to all siblings if the function $\psi(U_{i,1}, n_0)$ is increasing and concave, so that (II.1) can be rewritten as:

$$U_0 = v(c_0, n_0) + n_0 \psi\left(U_{i,1}, n_0\right) \tag{II.2}$$

Assuming that U_1 depends linearly on U_0, we could reformulate (II.2) as:

$$U_0 = v(c_0, n_0) + a(n_0) n_0 U_1 \tag{II.3}$$

[32] Note that the absence of subscripts to the altruistic function ψ lays on the assumption of equal value of siblings for parent's utility.

Where the term $a(n_0)$ measures the degree of altruism. The usual assumption is that for any given level of children utility U_1, the function U_0 in (II.3) is increasing and concave, i.e.:[33]

$$v_n + a(n_0) + n_0 a'(n_0) > 0, \quad v_{nn} + 2a'(n_0) + n_0 a''(n_0) < 0 \qquad \text{(II.4)}$$

As utility form is assumed to be identical for all cohorts, we can express the dynastic utility function as:

$$U_0 = \sum_{i=0}^{\infty} A_i N_i v(c_i, n_i) \qquad \text{(II.5)}$$

Where A_i is the implied degree of altruism of the dynastic head toward each descendant in the ith generation and N_i is the number of descendants in the ith generation:

$$A_0 = 1 \quad A_i = \prod_{j=0}^{i-1} a(n_i), \;\; i = 1,2, \ldots \qquad \text{(II.6)}$$

$$N_0 = 1 \quad N_j = \prod_{j=0}^{i-1} n_j\,, \;\; i = 1,2, \ldots \qquad \text{(II.7)}$$

A parent is "selfish" if the marginal utility of own consumption exceeds the marginal utility derived from his child's consumption when the parent has one child ($n = 1$). This definition implies that $a(1) < 1$ for selfish parents. We assume that parents are "selfish" because the utility of a dynastic family with stationary consumption per person ($c_i = c$) and a stationary number of descendants ($N_i = 1$) would be bounded only if $a(1) < 1$.

During adulthood, agents supply a unit of labour and receive a wage w_i; they leave a non-depreciable bequest k_{i+1} to any children n_i at the beginning of child's adulthood. Capital k_i earns a rent r_i. Assuming that the total cost of rising children is β_i, the intertemporal budget constraint is given by:

[33] Note that, with this specification, children could even provide disutility for a non-altruistic agent.

$$w_i + (1 + r_i)k_i = c_i + n_i(\beta_i + k_{i+1}) \tag{II.8}$$

Note that the cost of rising children β_i is independent on children quality, which in turn is measured by their consumption c_{i+1}, wage w_{i+1} and inheritance k_{i+1}; moreover, no restrictions are put on k_i, so that debt can be left to children, provided that the present value of debts is zero.[34] The maximization of (II.5) subject to (II.8) leads to a chosen path for c_i, k_{i+1} and N_{i+1} assuming the path for w_i, r_i and β_i as given.

A useful simplification of the problem is introduced by the assumption that fertility does not affect current period utility v, and this can be done by assuming a constant elasticity of the degree of altruism with respect to the number of children, i.e.:

$$a(n_i) = \alpha(n_i)^{-\varepsilon} \tag{II.9}$$

in this case the degree of altruism toward descendants in (II.6) depends only on the number of descendants in generation i, N_i, specifically $A_i = \alpha^i(N_i)^{-\varepsilon}$. The condition $0 < a(1) < 1$ translates into the new one $0 < \alpha < 1$, while the increase and concavity assumption on (II.5) brings to $0 < \varepsilon < 1$. Given (II.9), the dynastic utility function in (II.5) is reformulated as:

$$U_0 = \sum_{i=0}^{\infty} \alpha^i (N_i)^{1-\varepsilon} v(c_i) \tag{II.10}$$

Given a constant level of aggregate consumption $C_i = N_i c_i$, parents will have the incentive of giving birth to an additional child only in case the derivative of (II.10) with respect to N_i (holding constant N_j and c_j) is positive, i.e.:

$$\frac{\partial U_0}{\partial N_i} = \alpha(1-\varepsilon)(N_i)^{-\varepsilon} v\left(\frac{C_i}{N_i}\right) + \alpha(N_i)^{1-\varepsilon} v'\left(\frac{C_i}{N_i}\right)\left(\frac{C_i}{(N_i)^2}\right) > 0$$

$$= \alpha(N_i)^{-\varepsilon}\left[v\left(\frac{C_i}{N_i}\right)(1-\varepsilon) + v'\left(\frac{C_i}{N_i}\right)\left(\frac{C_i}{N_i}\right)\right] > 0$$

34 This assumption is fundamental to avoid the emergence of Ponzi Games, so that the value of debt increases faster than the interest rate as parents always leave a negative bequest k_i to offspring.

as $\alpha(N_i)^{-\varepsilon} > 0$, the condition we need is:

$$\sigma(c_i) < 1 - \varepsilon \tag{II.11}$$

Where $\sigma(c_i) = v'(c_i)c_i/v(c_i)$.

The maximization of (II.10) subject to (II.8) leads to the following Lagrangian function:

$$\mathcal{L}(c_i, N_{i+1}, k_{i+1}, \lambda_i) = \sum_{i=0}^{\infty} \alpha^i (N_i)^{1-\varepsilon} v(c_i) - \lambda_i[w_i + (1 + r_i)k_i - c_i - n_i(\beta_i + k_{i+1})]$$

the first set of FOCs can be taken by:

$$\frac{\partial\mathcal{L}(\cdot)}{\partial c_i} = \alpha^i (N_i)^{1-\varepsilon} v'(c_i) + \lambda_i = 0$$

$$\frac{\partial\mathcal{L}(\cdot)}{\partial c_{i+1}} = \alpha^{i+1} (N_{i+1})^{1-\varepsilon} v'(c_{i+1}) + \lambda_{i+1} = 0$$

$$\frac{\partial\mathcal{L}(\cdot)}{\partial k_{i+1}} = \lambda_i n_i - \lambda_{i+1}(1 + r_{i+1}) = 0$$

from the last equation:

$$\frac{\lambda_i}{\lambda_{i+1}} = \frac{(1+r_{i+1})}{n_i}$$

while from the first two:

$$\frac{v'(c_i)}{\alpha(n_i)^{1-\varepsilon} v'(c_{i+1})} = \frac{\lambda_i}{\lambda_{i+1}} = \frac{(1+r_{i+1})}{n_i}$$

So that the first set of FOCs is given by:

$$\frac{v'(c_i)}{v'(c_{i+1})} = \alpha(1 + r_{i+1})(n_i)^{-\varepsilon} \tag{II.12}$$

the second set of FOCs is easily derived by an alternative formulation of our Lagrangian equation:

$$\mathcal{L}(c_i, N_{i+1}, k_{i+1}, \lambda_i) = \sum_{i=0}^{\infty} \alpha^i (N_i)^{1-\varepsilon} v(c_i) - \lambda_i [k_0 + \sum_{i=0}^{\infty} d_i N_i w_i - \sum_{i=0}^{\infty} d_i (N_i c_i + N_{i+1} \beta_i)]$$

Where $d_i = \prod_{j=0}^{i} (1 + r_j)^{-1}$. Partial derivatives with respect to consumption and fertility are:

$$\frac{\partial \mathcal{L}(\cdot)}{\partial c_i} = \alpha^i (N_i)^{1-\varepsilon} v'(c_i) + \lambda_i d_i N_i = 0$$

$$\frac{\partial \mathcal{L}(\cdot)}{\partial N_i} = \alpha^i (1-\varepsilon)(N_i)^{-\varepsilon} v(c_i) + \lambda_i d_i [w_i - c_i - \beta_{i-1}(1 + r_t)] = 0$$

so that by substituting for the expression for $\lambda_i d_i$:

$$(1-\varepsilon) v(c_i) = v'(c_i)[w_i - c_i - \beta_{i-1}(1 + r_t)]$$

recalling the expression for $\sigma(c_i)$:

$$v(c_i)[1 - \varepsilon - \sigma(c_i)] = v'(c_i)[\beta_{i-1}(1 + r_t) - w_i] \quad \text{(II.13)}$$

finally we can derive the dynastic budget constraint by simply:

$$\frac{\partial \mathcal{L}(\cdot)}{\partial \lambda_i} = k_0 + \sum_{i=0}^{\infty} d_i N_i w_i - \sum_{i=0}^{\infty} d_i (N_i c_i + N_{i+1} \beta_i) = 0 \quad \text{(II.14)}$$

Note that (II.12) is an arbitrage condition for shifting consumption from one generation to the next; in particular the utility rate of substitution, $\frac{v'(c_i)}{v'(c_{i+1})}$, depends directly on the interest rate and the time-preference factor α, while it is negatively affected by the number of children at time i.[35]

[35] The interpretation for this relationship is that, given the altruism coefficient α, an increase in n_i decreases the altruism factor $\alpha^i (N_i)^{1-\varepsilon}$, enhancing the discount rate of future consumption.

Equation (II.13) equates the marginal benefit of an additional child to the marginal cost, while holding fixed the total consumption C_i of that generation. As discussed earlier, this marginal utility must be positive near an optimal position, which implies that $1 - \varepsilon - \sigma(c_i) > 0$ by equation (II.11). The same condition is found by SOCs for maximization. Just computing the Hessian Matrix of the Lagrangian:

$$\begin{pmatrix} \frac{\partial^2 \mathcal{L}(\cdot)}{\partial {c_i}^2} & \frac{\partial \mathcal{L}(\cdot)}{\partial c_i \partial N_i} & \frac{\partial \mathcal{L}(\cdot)}{\partial c_i \partial \lambda_i} \\ \frac{\partial \mathcal{L}(\cdot)}{\partial N_i \partial c_i} & \frac{\partial^2 \mathcal{L}(\cdot)}{\partial {N_i}^2} & \frac{\partial \mathcal{L}(\cdot)}{\partial N_i \partial \lambda_i} \\ \frac{\partial \mathcal{L}(\cdot)}{\partial \lambda_i \partial c_i} & \frac{\partial \mathcal{L}(\cdot)}{\partial \lambda_i \partial N_i} & \frac{\partial^2 \mathcal{L}(\cdot)}{\partial {\lambda_i}^2} \end{pmatrix}$$

Where:

$$\frac{\partial^2 \mathcal{L}(\cdot)}{\partial {c_i}^2} = \alpha^i (N_i)^{1-\varepsilon} v''(c_i) < 0$$

$$\frac{\partial \mathcal{L}(\cdot)}{\partial c_i \partial N_i} = \frac{\partial \mathcal{L}(\cdot)}{\partial N_i \partial c_i} = \alpha^i (1-\varepsilon)(N_i)^{-\varepsilon} v'(c_i) - \lambda_i d_i = 0$$

$$\frac{\partial^2 \mathcal{L}(\cdot)}{\partial {N_i}^2} = \alpha^i (1-\varepsilon)(-\varepsilon)(N_i)^{-\varepsilon-1} v(c_i) < 0$$

$$\frac{\partial \mathcal{L}(\cdot)}{\partial c_i \partial \lambda_i} = \frac{\partial \mathcal{L}(\cdot)}{\partial \lambda_i \partial c_i} = -d_i N_i < 0$$

$$\frac{\partial \mathcal{L}(\cdot)}{\partial \lambda_i \partial N_i} = \frac{\partial \mathcal{L}(\cdot)}{\partial N_i \partial \lambda_i} = d_i [w_i - c_i - \beta_{i-1}(1 + r_t)]$$

$$\frac{\partial^2 \mathcal{L}(\cdot)}{\partial {\lambda_i}^2} = 0$$

The signs are associated according to assumptions on equation (II.10) and the FOCs in (II.12) and (II.13). Hence, in order to find a maximum in the optimization process, we need the Lagrangian function to be concave. The condition on pure derivatives is satisfied, as $\frac{\partial^2 \mathcal{L}(\cdot)}{\partial {c_i}^2}$, $\frac{\partial^2 \mathcal{L}(\cdot)}{\partial {N_i}^2}$, $\frac{\partial^2 \mathcal{L}(\cdot)}{\partial {\lambda_i}^2}$ are all non-positive; however, the condition for the definition of the Hessian Matrix is needed, so that:

$$\left[\alpha^i(1-\varepsilon)\varepsilon(N_i)^{-\varepsilon-1}v(c_i)\right][d_iN_i]^2+\{d_i[w_i-c_i-\beta_{i-1}(1+r_t)]\}^2\alpha^i(N_i)^{1-\varepsilon}v''(c_i)<0$$

After few algebraic passages:

$$(1-\varepsilon)\varepsilon v(c_i)+[w_i-c_i-\beta_{i-1}(1+r_t)]^2v''(c_i)<0$$

Just applying the expression for the second member from partial derivative with respect to fertility:

$$\varepsilon+\frac{v(c_i)(1-\varepsilon)}{[v'(c_i)]^2}v''(c_i)<0$$

If we assume a CES form for $v(c_i)$, i.e. $v(c_i)=(c_i)^\sigma/\sigma$, then we will have:

$$\varepsilon+\sigma-1<0$$

Which coincides with equation (II.9).

In particular, condition (II.11) appears to imply that consumption is positive only when children are a financial burden, that is when the cost of rearing a child exceeds the present value of his lifetime earnings.

Just applying the definition of $\sigma(c_i)$ to (II.13):

$$c_i[1-\varepsilon-\sigma(c_i)]/\sigma(c_i)=\beta_{i-1}(1+r_t)-w_i \tag{II.15}$$

The optimal solution for the fertility rate n_i can be found simply by substituting (II.15) into (II.12) and rearranging:

$$n_i=\left[\alpha(1+r_{i+1})\frac{v'(c_{i+1})}{v'(c_i)}\right]^{\frac{1}{\varepsilon}}=\left[\alpha(1+r_{i+1})\frac{\beta_{i-1}(1+r_t)-w_i}{\beta_i(1+r_{t+1})-w_{i+1}}\right]^{\frac{1}{\varepsilon}} \tag{II.16}$$

2.2.2 The Production side

In an open economy, where the interest rate is given exogenously by the rest of the world, a country's aggregate holding of assets, K_i, could differ from its stock of productive capital. Instead, we assume a closed economy where the holding of assets equals the stock of productive capital and, as in standard growth models of a closed economy, interest rates and wage rates, r_i and w_i.

We specify that production takes place in "firms" via a standard one-sector production function that exhibits constant returns to scale and Harrod-neutral technical progress.[36] Hence:

$$Y_i = F[K_i, (1+g)^i L_i] \tag{II.17}$$

Where Y_i is the total level of output, K_i is the level of physical capital in the economy, L_i is the labour force and g represents the exogenous technological progress. Letting $\hat{y}_i = Y_i/(1+g)^i L_i$ and $\hat{k}_i = K_i/(1+g)^i L_i$, (II.17) can be rewritten as:

$$\hat{y}_i = f(\hat{k}_i) \tag{II.18}$$

$$f' > 0\,, f'' < 0$$

Assuming competitive markets and profit maximization, factors will be paid according to their production shares, so that:

$$r_i = f'(\hat{k}_i) \tag{II.19}$$

$$w_i = [f(\hat{k}_i) - \hat{k}_i f'(\hat{k}_i)](1+g)^i \tag{II.20}$$

On the side of households, it is assumed that each child requires both time and material goods in order to be reared. In this sense, its bearing cost can be represented in the following form:

[36] This assumption is necessary in order to deal with steady states where control variables grow at a constant rate.

$$\beta_i = a(1+g)^i + bw_i \tag{II.21}$$

$a \geq 0\,, 0 \leq b < 1$

2.2 Comparative statics

Assuming that $\sigma(c_i)$ grows less than c_i, we could say that the left hand side of (II.15) is increasing in c_i, so that c_i is a positive function of the net cost of producing another descendant in generation i; in other words, consumption per person, c_i, would rise across generations only if the net cost of creating descendants also rose. This constitutes a fundamental difference with usual conditions in optimal consumption models.[37] This result derives from the fact that, this in specification, the rate of growth across generations of consumption per person is essentially independent on the level of interest rates, and also does not depend on pure altruism or time preference.

Regarding fertility rate, n_i, we could say that, according to equation (II.16), it rises with the altruism coefficient α and the interest rate r_{i+1}; while the first dependence, the one on α is expectable, the other is not as straightforward. This result is derived from the consideration that there is an effect implied by the interest rate that tends to increase consumption over time and this effect dominates the increase in the cost for capital in the steady state.

Another important property of the model concerns the effects of changes in wealth, which we represent by shifts in the initial asset level k_0. Equation (II.15) implies that future consumption per person, c_i, is unaffected if a shift in wealth does not change the net cost of raising children. Then equation (II.16) implies that future fertility n_{i+1}, for $i = 1,2,\ldots$, also does not change. With future consumption per capita and future fertility unchanged, the dynastic budget equation (II.14) requires either initial consumption c_0 or fertility n_0 to change. Using equation (II.16) for $i = 0$, we can see

[37] The most common condition for consumption growth over time is that the rate of time preference, in our case the altruism factor, is lower than the real interest rate.

that an increase (or decrease) in c_0 must be accompanied by an increase (or decrease) in n_0.

These results imply that an increase in inherited wealth increases only the scale of a dynastic family. The number of descendants, N_i, and aggregate consumption C_i in each future generation would increase only because of the increase in initial fertility n_0. To see the effect on N_i directly, the substitution for each fertility rate from equation (II.16) leads to:

$$N_i = \left[\alpha^i \frac{v'(c_i)}{v'(c_0)} \prod_{j-1}^{i} (1 + r_j)\right]^{\frac{1}{\varepsilon}} \quad i = 1,2, \ldots \tag{II.22}$$

so that an increase in c_0 determines an increase in N_i as all future values of consumption will be unchanged.

A somewhat surprising result is that future capital per person, k_i for $i = 1, 2, \ldots$, is not affected by a change in wealth. This result follows from the dynastic budget constraint in equation (II.14) because future consumption per person, c_i, and fertility, n_i, are unchanged. Put differently, bequests to each child are unaffected by a change in parent's wealth.

The model also has surprising implications about the effects of temporary changes in the cost of producing children β at time i on fertility, compensated by a subsidy on wealth k_0, so that the marginal utility $v'(c_i)$ remains unchanged. While (II.15) indicates that c_i rises and consumption in other periods do not change, equation (II.16) implies that n_i falls; however n_{i+1} rises exactly in the same proportion to offset the fall in n_i. The reason is that dynastic utility in equation (II.10) is a time-separable function of the number of descendants and consumption in each generation. Dynastic utility does not depend explicitly on the fertility of any generation.[38]

Consider now a compensated, permanent increase in the cost of children that raises the net cost of children, $\beta_i\,(1 + r_{i+1}) - w_{i+1}$, by the same proportion for each

[38] Time-separable utility functions imply that the demand for a variable at time i depends only on the marginal utility of wealth and the prices of variables at time i. Consequently, for a given marginal utility of wealth, the number of descendants and consumption in generation i would not be affected by price changes in other generations.

generation $i \geq j$. According to (II.16), we know that consumption in any generation $i \geq j$, c_i, will increase, and this shift will be equiproportional if we assume that the elasticity of $v(c_i)$ with respect to c_i is constant and equal to σ; hence (II.12) simplifies in the following form:[39]

$$\left(\frac{c_{i+1}}{c_i}\right)^{1-\sigma} = \alpha(1 + r_{i+1})(n_i)^{-\varepsilon} \tag{II.23}$$

So that fertility in j, n_j, falls as $\frac{c_{j+1}}{c_j}$ rises, but it will remain unchanged for all subsequent periods $i > j$; though this is only a temporary effect on the relevant decision variable, fertility, this will affect also subsequent generations as the number of descendants will be inevitably lower. Moreover, higher values of capital endowment per person, k_i, are needed in order to support higher levels of consumption in all periods $i > j$.

Another important issue to consider is the effect of child mortality. To tackle this topic, just assume that wage rates and interest rates are stationary over time, and that parents ignore the uncertainty about child deaths and respond only to changes in the fraction p of offspring that survive childhood. Let β_s be the constant marginal cost of rearing a child to adulthood, and β_m be the cost of a child that dies prior to becoming an adult. The expected cost of n_b births is given by $p\beta_s + (1-p)\,\beta_m$. The ratio of this expected cost to the expected number of survivors ($n = pn_b$) which corresponds to our previous cost per (surviving) child is:

$$\beta = \beta_s + \beta_m\,(1-p)/p \tag{II.24}$$

A permanent decline in the level of child mortality lowers the cost of raising surviving children in all generations. Our prior analysis implies that the demand for surviving children per adult (n_i) rises in the initial generation, but that it is no higher in later generations. Since the demand for surviving children increases in the initial generation,

[39] Assuming that the elasticity of $v(c_i)$ with respect to c_i is constant and equal to σ is equivalent to assume that the function $v(c_i) = (c_i)^\sigma/\sigma$.

birth rates may also rise then, although the higher probability of survival, p, reduces the number of births, n_b, needed to produce a given number of survivors. Birth rates definitely fall in later generations because the demand for surviving children in these generations would not be affected by the increase in p.

If child mortality continues to fall over time, the cost of rearing surviving children would continue to fall over time, and hence the demand for surviving children per adult would increase for more than one generation. However, the rate of decline in child mortality must slow down once it approaches zero, as it has in the West during the past forty years. As the rate of decline slows, the rate of decline in the cost of producing survivors also slows and eventually more or less ceases. Thereafter, the cumulative increase in the child survival probability does not affect the demand for surviving children.

The model is not set up to incorporate social security precisely because we have only one period of adulthood; as previous Chapter shows, at least three periods (childhood, adulthood and seniority) are necessary for considering fertility choices and social security systems. Therefore, a pay as- you-go system of taxes on young working adults cannot finance payments to old adults. However, similar results can be obtained if we imagine (unrealistically) that a tax is levied on children to finance transfers to adults. This assumption can be compared to the standard hypothesis for social security (that a tax is imposed on working age wage to collect resources to be redistributed to retired individuals) to the extent of anticipating the contribution of the young cohort in favour of the old. However, this setting would make the child birth giving and the advantages from the social security system (transfers to adults) concide. Let s_i be the transfer received by the representative adult in generation i, and τ_{i+1} be the tax paid during generation i by each child (or by parents on behalf of their children). The government's budget is balanced if $s_i N_i = \tau_{i+1} N_i$, which implies that:

$$\tau_{i+1} = s_i / n_i \qquad \text{(II.25)}$$

For given values of fertility, the benefits from social security and the taxes to finance them have exactly offsetting effects on the dynastic wealth of the representative family. Therefore, if fertility were unchanged, a change in the scale of the social security program would not affect intergenerational patterns of consumption. Parents would raise their bequests sufficiently so that their children can pay these taxes without cutting back on their consumption (see Barro [1974]).

However, the endogeneity of fertility modifies the so-called "Ricardian Equivalence Theorem." The social security program imposes the lifetime cost per child of:

$$\frac{s_i}{n_i} - \frac{s_{i+1}}{1+r_{i+1}} \tag{II.26}$$

Given a flat subsidy system, that is $s_i = s = s_{i+1}$, the net tax is positive if $1 + r_{i+1} > n_i$. With a positive net tax, an increase in the scale of the social security program (an increase in s) raises the cost of children; the same substitution effect as an increase in the cost of raising a child, β. Therefore, our previous analysis of the effects of changes in the cost of children applies to social security. Therefore, a permanent increase in social security benefits tends to reduce fertility temporarily even when children do not support their elderly parents.

The same holds for the impact on consumption; the positive effect of higher social security benefits on the cost of rearing children would raise "capital intensity".

2.3 The Equilibrium

The time allocated to work L_i depends on the number of adults in the cohort N_i and the time allocated to rising children b; in particular, parents should devote bn_i units of time for child rearing, so that the labour supply curve is defined by:

$$L_i = (1 - bn_i)N_i$$

So that dividing equations by $(1 - bn_i)$ permits to express variables in terms of labour input units. After doing so to the intertemporal budget constraint in (II.5), and expressing in efficiency units (so that dividing by $(1 + g)^i$), we get:

$$\widehat{w}_i + (1 - bn_i)(1 + r_i)\hat{k}_i = \hat{c}_i + n_i\left[\hat{\beta}_i + (1 + g)(1 - bn_{i+1})\hat{k}_{i+1}\right] \qquad \text{(II.27)}$$

Where $\widehat{w}_i = w_i/(1 + g)^i = [f(\hat{k}_i) - \hat{k}_i f'(\hat{k}_i)]$, $\hat{\beta}_i = \beta_i/(1 + g)^i = a + b\widehat{w}_i = a + b[f(\hat{k}_i) - \hat{k}_i f'(\hat{k}_i)]$ and $\hat{c}_i = c_i/(1 + g)^i$. By dividing equation (II.15) by $(1 + g)^i$, we can get:

$$\hat{c}_i = \left(\frac{\sigma}{1-\varepsilon-\sigma}\right)\left[\hat{\beta}_{i-1}\left(\frac{1+r_i}{1+g}\right) - \widehat{w}_i\right] \qquad \text{(II.28)}$$

By substituting (II.28) into (II.27) we can get the fundamental function for analysing the dynamics of physical capital per efficient worker accumulation:

$$\widehat{w}_i + (1 - bn_i)(1 + r_i)\hat{k}_i = \left(\frac{\sigma}{1-\varepsilon-\sigma}\right)\left[\hat{\beta}_{i-1}\left(\frac{1+r_i}{1+g}\right) - w_i\right] + n_i\left[\hat{\beta}_i + (1 + g)(1 - bn_{i+1})\hat{k}_{i+1}\right] \qquad \text{(II.29)}$$

And finally, by substituting (II.27) into (II.16), we can get:

$$n_i = \left\{\alpha(1 + r_{i+1})\left[\frac{1}{(1+g)}\frac{\hat{\beta}_{i-1}(1+r_t)-(1+g)\widehat{w}_i}{\hat{\beta}_i(1+r_{t+1})-(1+g)\widehat{w}_{i+1}}\right]^{(1-\sigma)}\right\}^{\frac{1}{\varepsilon}} \qquad \text{(II.30)}$$

In the steady state $\hat{k}_i$ does not change; as a consequence, even $n_i, \hat{c}_i, r_i, \widehat{w}_i$ and $\hat{\beta}_i$ are constant in equilibrium, so that from (II.27), (II.28), (II.29) and (II.30) we can get:

$$\widehat{w} + (1 + r)\hat{k} = f(\hat{k}) + \hat{k} = \hat{c} + n\left[\hat{\beta} + (1 + g)\hat{k}\right] + bn\hat{k}[1 + r - n(1 + g)] \qquad \text{(II.31)}$$

$$\hat{c}_i = \left(\frac{\sigma}{1-\varepsilon-\sigma}\right)\left[\hat{\beta}\left(\frac{1+r}{1+g}\right) - \widehat{w}\right] \qquad \text{(II.32)}$$

$$f(\hat{k}) + \hat{k} = \left(\frac{\sigma}{1-\varepsilon-\sigma}\right)\left[\hat{\beta}\left(\frac{1+r}{1+g}\right) - \widehat{w}\right] + n\left[\hat{\beta} + (1 + g)\hat{k}\right] + bn\hat{k}[1 + r - n(1 + g)] \qquad \text{(II.33)}$$

$$n = \left[\frac{\alpha(1+r)}{(1+g)^{(1-\sigma)}}\right]^{\frac{1}{\varepsilon}} \qquad \text{(II.34)}$$

2.4 Discussion

The second Chapter develops the Barro-Becker (1989) model as an example of intergenerational model with altruism. In this framework, agents live for two periods, but the altruism toward children permits to consider the possibility of infinitively lived agents maximizing instantaneous utilities according to a discount factor. A peculiarity of the model is the concept of pure altruism, by which agents are interested to descendants' utilities and not other measures (for instance child quantity). This altruism function is assumed to be a decreasing function of fertility itself, but still utility is increasing and concave in offspring number; the same happens for consumption. A fundamental and very strong assumption made in the model is the additivity and separability of utility, which stands for time consistency of choices. Though this assumption could result fuzzy in an intergenerational model (the idea that generations will be motivated always and only by altruism, without deviations from a sort of "social contract") it is a logic consequence of homogeneity assumptions. If generations are all equal among themselves, and differ only because of cohort size, this fundamental condition is surely justified. Moreover, the production side is assumed to replicate the Solowian framework. Conclusions that are drawn from this framework are, to some extents, in line with what proposed by approaches studied in the first Chapter of this work. Consumption is a positive function of the net cost of producing another descendant in the same generation. This result derives from the fact that, this in specification, the rate of growth across generations of consumption per person is essentially independent of the level of interest rates, and also does not depend on pure altruism or time preference. On the other side, fertility rises with parental altruism coefficient and the interest rate (same conclusion as in the case of an economy with social securities). This result is derived from the consideration that there is an effect implied by the interest rate that tends to increase consumption over time and this effect dominates the increase in the cost for capital in the steady state. A rise in the initial level of wealth will only have an effect in the short run, with an increase in both consumption and fertility, but it will disappear in the long run. Given that wealth is a component of income per capita (as it can be reinvested in capital accumulation and labour force through fertility), this hump shaped behavior of fertility can be interpreted as the model answer to demographic transition. The introduction of child mortality will decrease (surviving) child rearing cost and so that demand for children; this result is in line with previous literature.

However, other results seem to give an answer to some of previous contradicting conclusions. For instance, the introduction of a social security system will activate a substitution mechanism that will tend to reduce fertility in the long run, and this in line with Prinz (1990) and Zhang (1995); this condition will hold depending on the interest rate and fertility. However, the opposite condition will be triggered if the condition on parameters does not hold, and this validates Cigno and Rosati (1992). Parents would not change consumption patterns (as dynastic wealth has not changed) and will generate less children because of an increase in the cost of rising them (consequence of the Ricardian Equivalence).

Chapter 3: An Empirical Insight

The last Chapter of this Thesis is devoted to the study of the empirical relationship between income growth and fertility choices. Income is the main source of structural change in a society: greater economic capabilities tend to increase human capital investment (better or more education), decrease child mortality and shift social security systems from a "children supporting parents" setting to a modern pension system. In simple terms, it boosts all those mechanisms described in the first Chapter (the quality-quantity tradeoff, the child and infant mortality issue and the old age security hypothesis) apt at recreating the so called demographic transition.

In fact, many authors challenged this empirical trend. Among the earliest works Coale and Hoover (1958), suggested that high fertility hampers large real per capita income growth; this conclusion supports the so called "Neo-Malthusian" theory. One of the most common hypotheses for this phenomenon lies on technological arguments. The reason is that technical progress being non-rival, the cost of inventing new technologies is independent of the number of individuals who use it: there is a scale effect. That is, for a constant share of resources allocated to the development of new technologies, a larger population stimulates the rate of technological progress, so the rate of income growth (see Kremer, 1993). This conclusion is generally shared by semi-endogenous growth models built on economies characterized by an R&D sector (Jones, 1995; Kortum, 1997; Segerstrom, 1998).

However, some other works did not support this positive correlation between income per capita and fertility (see McNicol, 1984). The ensuing literature can be roughly divided into two strands. One approaches the issue from a theoretical point of view and finds that, interpreted or with the appropriate additions in choice variables, fertility should be negatively related to income. The basic idea is that the price of children is largely time, and because of this, children are more expensive for parents with higher wages. Another argument is that higher-wage people have a higher demand for child quality, making quantity more costly, and hence those parents want fewer children. The other strand of literature approaches the issue from an empirical point of view, arguing that the negative relationship is mainly a statistical fluke due to a missing variables problem. This literature focuses on identifying those crucial missing variables, such as female earnings potential. Once those missing variables are controlled for, fertility and income are actually positively related (ee Hotz, Klerman, and Willis, 1993).[40] No doubt, this hypothesis has received a much larger support than the "Neo-Malthusian" one (among all Jones and Tertilt, 2008).

The difficulties in extending and generalizing experiments on single country or group of countries to Continents or to the World led the scientific community to begin to give an ambivalent answer to the question of the relation between GDP and population measures (see Kelley, 1988).[41] Among all, Andersson, Kreyenfeld and Mika (2009) showed how, even controlling for female earnings, the relationship between income and fertility may turn up to be uncertain. Though Denmark and West Germany may seem to be similar countries in terms of economic fundamentals and traditions, they manage to find an opposite impact on fertility because of changes in women wages.

With the precise aim of giving an answer to this puzzle, we will dedicate the rest of this work to data analysis. The rest of the Chapter is organized as follows. The first paragraph gives a complete framework of what has characterized very long run trends in population and income from the nineteenth Century to nowadays. A selection of 12 countries from 4 Continents will constitute a sample for examining the relation between income and population on different stages of economic development. The second Paragraph will concentrate on a complete Regional (Continental) analysis on the population-income relation; this experiment will be particularly useful to understand the demographic transition timing among different Geographical and Economic Regions.

[40] However, the negative effect of female wages can be mitigated or even reversed by the link between parental benefits and previous earnings. It is reasonable to expect that the system encourages women to find a job before having a child even if they are planning to stay at home for a relatively long time (Hoem, 2000).

[41] Some authors (Simon, 1989) ended up arguing that the failure in proving a negative effect of population growth on income per capita may be interpreted as the absence of this relation.

3.1 A Global Analysis

As already mentioned, this Paragraph will intensively tackle the empirical issue of the relation between income and fertility choice. The focus of our analysis will comprehend two entire Centuries (from 1870 to 2008) for 12 countries; this choice is aimed at weighing the sample in order to be fairly representative of what the World has experienced in the last decades. For a complete list of selected countries, see Appendix A. Unfortunately, the inclusion of African countries inside the sample would create serious representation difficulties. The availability of data for this Region is so low that no long run interesting analysis can be made with all Continents. The inclusion of Africa in this Global Study will certainly delate income phenomena and the demographic transition of what we call "The World".[42] We will both concentrate on what our international sample has experienced in this very long (run) period and the dynamics in single Continents. Though this can't be considered a complete representation of all geographical Areas, we will call it the World for simplicity sake.

The aim of this study is to understand whether and when the demographic transition has produced its effects on population dynamics at an international level. Demographers study this phenomenon on a different side (see for instance Bongaarts, 2010), in general through the behaviour between fertility rates (i.e. crude birth rate) and mortality. During the transition, first mortality declines and then fertility follows, causing population growth rates first to accelerate and then to slow again, moving toward low fertility, long life and an old population. However, in economics, it is much more common and useful to understand how population growth depend on measures of income (see Jones and Tertilt, 2006; Galor, 2012). In this sense, we adopt general income measures (Total GDP and GDP per capita) and demographic indexes (as total population or population growth rate).

In order to understand the long run trend in demographic and economic structures, it is useful to investigate data on income and population at an aggregate level. In this sense, this paragraph is devoted to depicting time series of our international sample's

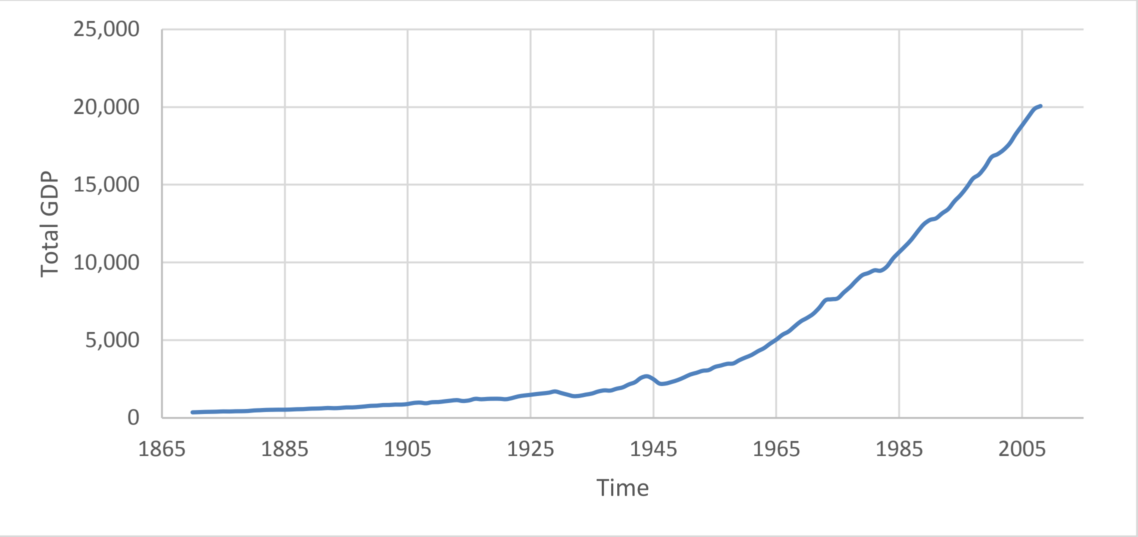
[42] Africa has been the last Continent experiencing the demographic transition. Moreover, it is the second most populous (and the first in terms of population growth rate) and the one with the lowest GDP per capita (and GDP per capita growth rate) Area considered in this study. Its inclusion in the international sample would increase population growth rate and decrease income per capita after the 50's, in such a way that the ongoing transition would be shadowed.

income (both in total and in per capita terms) and population.

3.1.1 Data

During the period of our analysis, the World experienced a stable and sustained growth of Total Output. While in 1870, average income among countries was worth 359,363 bilions 1990$, while in 2008 it grew up to 20,053,239 trilion 1990$, about 58 times larger. However, the rate of growth of income is clearly not constant. In fact, we can see total income trends in Figure 1:

Figure 1: Total GDP for the international sample in the period 1870-2008, in trillions 1990$

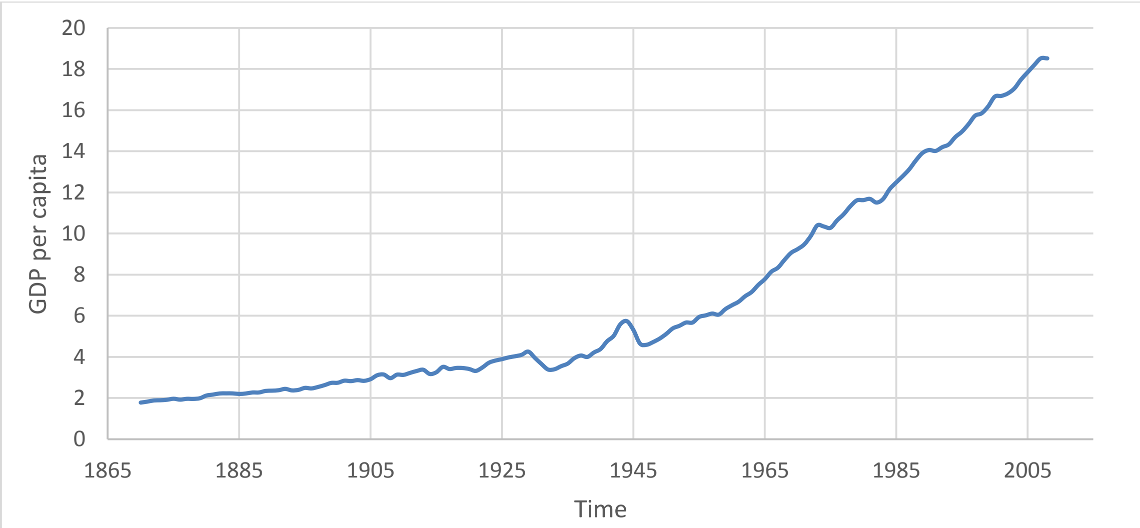


Source: *Maddison (2013)*

Income has grown unevenly among times. In particular, the first two decades of our analysis showed an annual growth rate of total output of about 2.5%, followed by an annual rate of 2%, and 2.8% until the 50's. After the Second World War, the "Global" Takeoff permitted countries to grow at annual rates near the 4.3% (as during the Economy Boom, boosted by the Marshall Plan), but after this decade, income growth steadily declined to 2.5%. As we will see, this phenomenon has been a joint effect of a decrease in productivity (GDP per capita growth) and population growth.

On the side of income per capita, the increase over the period considered in this analysis is much lower than that observed for total income (from 1669.7 to 17421.9 1990$, about 10 times larger). However, as for total output, even per capita GDP has shown some very different rate of increase through time, as we can see from Figure 2:

Figure 2: Real GDP per capita for the international sample in the period 1870-2008, in thousands 1990$

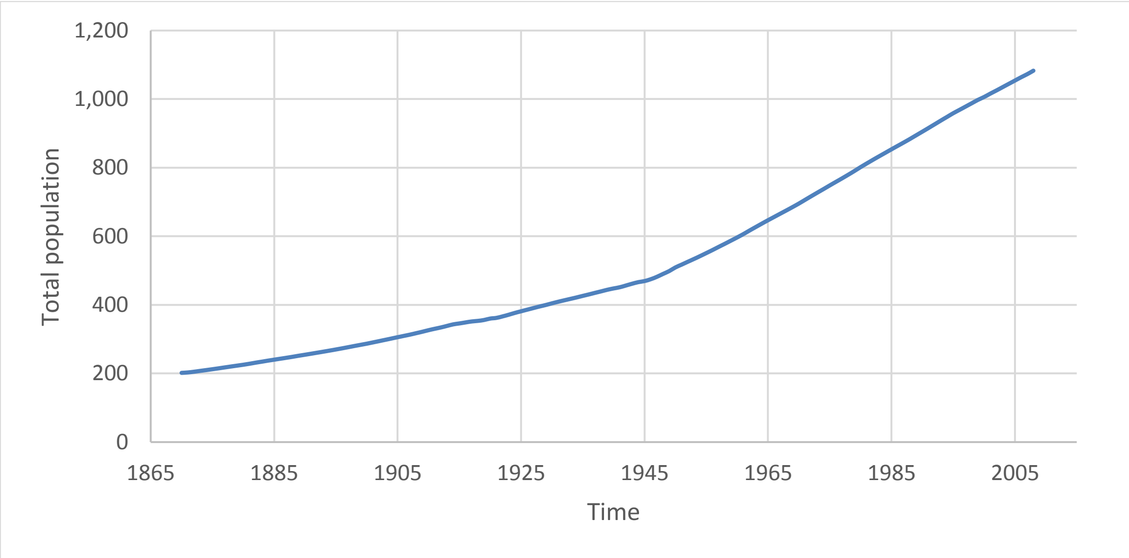


Source: *Maddison (2013)*

All in all, the annual growth rate of income per capita has been relatively proportional with the growth rate of total income. In fact, annual per capita GDP growth rate has decreased from 1.28% and 1.37% in 1871-1890 and 1891-1910 ("Liberal" order) to 1.02% in 1911-1930 (mainly caused by First World War and the collapse of trade and capital markets). However, war expenditure during the Second World War permitted a rapid increase of income per capita until the late 40's, with an annual rate of 1.7%. Nevertheless, a great upturn of GDP per capita growth took place during the Economic Boom, reaching a growth rate of 2.7% in 1951-1970 (the Maddison's "Golden Age"), 2% and 1.56% in 1971-1990 and 1991-2008 (also called the "Neo-liberal" order). This behaviour would constitute the basis for the transition from a state of high population growth and stagnant income to a reversed situation for most of the Second World countries (e.g. East Asia and Latin American countries).

On the population side, turning our attention to the sample considered above, we could detect a similar behaviour of the one characterizing income measures. In fact, as for output, total population has considerably increased in the period of analysis (it grew from 201,671 milions to 1,082,943 billions, about 5.4 times),[43] but the rate of growth has changed in different periods; for instance, annual population growth rate has increased from 1.1% in the period 1871-1950 to 1.5% in 1951-1970, then decreased to 1.25% and 1% in the period 1971-1990 and 1991-2008.

Figure 3: Total population for the international sample in the period 1870-2008, in millions



Source: *Maddison (2013)*

3.1.2 The World's Demographic Transition

As already mentioned, the demographic transition is analysed with particular focus on the relation between income and population measures. As already suggested by Galor (2005), we will conduct our analysis using per capita GDP and population (both in levels and in growth rate terms). Moreover, according to Becker (1981), the decline in (fertility) population growth in the course of the demographic transition is a by-product

[43] The inclusion of other regions in the dataset (e.g. East Europe and Africa) would significantly change figures. Population would amount to more than 6.7 billion in 2008 and would have grown of more than 6 times since 1870.

of the rise in income per capita that preceded the demographic transition. In general, this theory claims for the necessity of a compositional analysis. The rise in income induces a (fertility) population growth decline via the positive income effect that was generated by the rise in wages, which is dominated by the negative substitution effect brought about by the rising opportunity cost of children (see models proposed in Chapter 1).

As already evident from other analysis (e.g. Madison 2003; Galor, 2005), three main periods characterizing the relation between income and population can be inferred; a Malthusian Epoch, a Post-Malthusian period and a Modern Regime. This is the result we aim to achieve in this Chapter; we focus our analysis in order to delimit a time period in which the demographic transition took place. However, in order to do this, we need to deeply understand what these three phases of human societies are characterized by.

In general, the Malthusian Era is characterized by high population growth and stagnant income per capita. Most of the authors (from Maddison to Jones) found this empirical regularity satisfied before the Twentieth Century, so that it is generally associated with the earliest stages of development. The “child and infant mortality” and the “social security” theory give an explanation to this regularity. Agents choose to have many children as mortality is relatively high and utility depends on surviving children. In the same way, people living in an agricultural society, choose to have children in order to ensure their old age consumption.

However, the standard definition of the Malthusian Era would complicate this analysis in an inconvenient way. In fact, in order to study the relationship on the side of fertility, we should study also the relationship with death rates (see for example, Bloom and Canning, 2001; Kalemli-Ozcan, 2002; Dyson, 2010);[44] this analysis would be needed in order to study the composite effect on population growth rate. This is, generally, what demographers do.[45] In this sense, according to Malthus’ theory, the Malthusian era is that period of human history in which total GDP grows only because

[44] The basis of these theories lye on the concept of intertemporal utilities. The more agents live, the more they will invest in education, physical capital and technological advancements (with a consideration to risk aversion); this postponement of utilities will permit individuals to acquire a higher income in the future because of wages increase due to higher schooling attainments and saving rates. This process will trigger the fertility channel afterwards.

[45] This approach is certainly more complete, as it discovers compositional effects between fertility and mortality rate, but even more it is capable of delineating the timing of such an interaction. However, our scope is much more limited, so that we use results on empirical literature to only address the issue on the side of population growth.

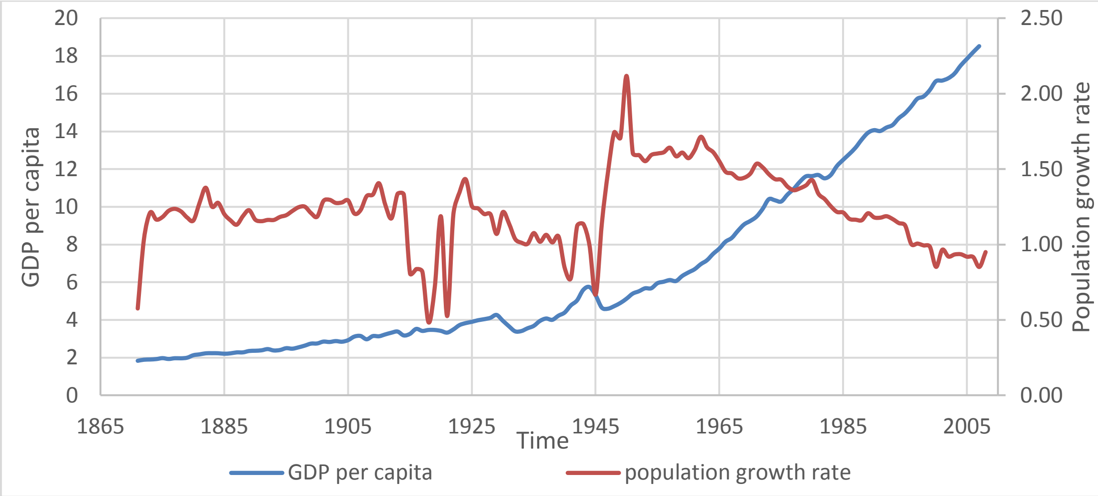
of its scale component, the population; this conclusion means that technological progress does not give any contribution to economic growth. As a consequence, the only relevant engine of growth is the labour force,[46] and per capita GDP should be nearly stagnant. The only enhances in GDP per capita would be triggered by land discoveries and would be consistent only in the short run, as it would be absorbed by population growth.

Though this theory is powerful in explaining human society development and demographics for several centuries, it needs a criterion to be tested. The latter is offered by correlation analysis. In fact, diminishing labour productivity and the positive effect of standard of living on population size kept income per capita at a subsistence level. Periods characterized by the absence of changes in the level of technology or in the availability of land, were characterized by a stable population size and constant income per capita, whereas periods characterized by improvements in the technological environment or in the availability of land generated small gains in income per capita. In this sense, as long as the correlation between income per capita and population is positive, then the Malthusian regime holds; when this regularity brakes, we face the demographic Transition.

We can test the World demographic transition both graphically and quantitatively. By plotting Figure 2 and the growth rate of population in Figure 3 we can see that the World has experienced the demographic transition in the 50's:

[46] In this context it is relatively forward to think that no countries are expected to undergo Demographic Transition before the Industrial Revolution has completely spread in Europe (see Ashfar and Galor, 2008). In fact authors find that the from first societies on Earth to the late Nineteenth Century, the Malthusian model has ruled income and population phenomena.

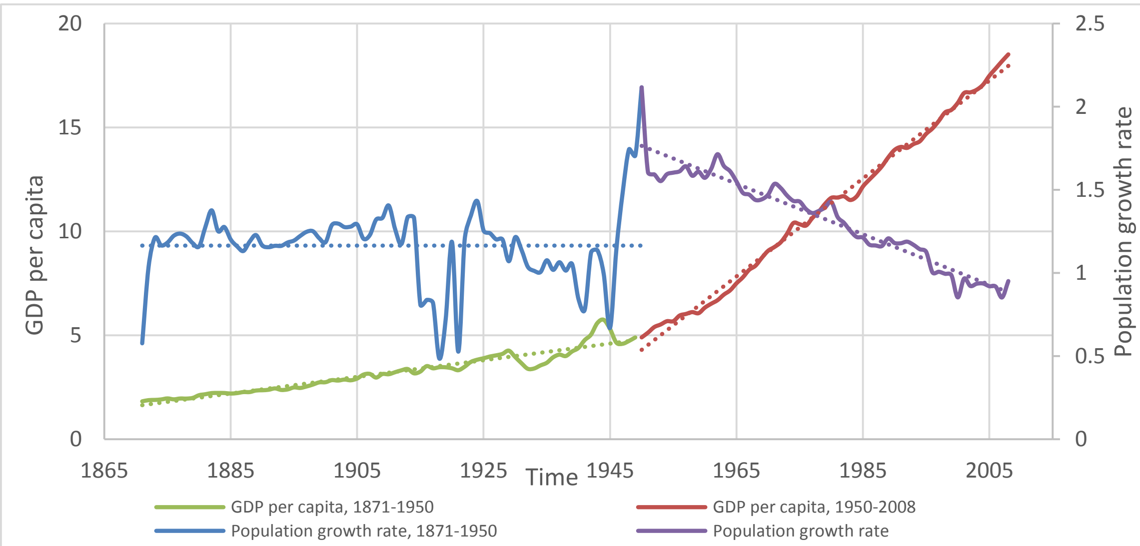
Figure 4: GDP per capita, in thousands 1990$, and population growth for the international sample, in the Period 1871-2008



Source: *Maddison (2013)*

Correlation coefficients can be studied by plotting GDP per capita and population growth rate in two distinct series, as in Figure 5:[47]

Figure 5: GDP per capita, in thousands 1990$, and population growth for the international sample, in the Period 1871-2008



Source: *Maddison (2013)*

[47] African countries are not accounted in this analysis. The inclusion will change results for its high population growth rate. As we will prove, Africa has experienced the Demographic Transition around the 90's, so that the population growth rate would have been increasing in the period 1950-1990.

The correlation analysis validates our graphical intuition. In fact, population growth rate and income per capita are positively correlated until the 50's with a coefficient of 0.015, while this relation becomes negative afterwards, with a coefficient of -0.97.

The following table (Table 1) gives a further proof of the demographic transition. Correlation coefficients between GDP per capita and population growth rates are computed. On the first column we report time periods, while on the second we display correlation coefficients and 95% confidence intervals in parentheses. Long run time periods are reported in bold, together with correlation coefficients (with 95% confidence intervals).

Table 1: Correlation coefficients between per capita GDP and population growth rate for the World, 1871-2008. Confidence Intervals at 95% in parentheses

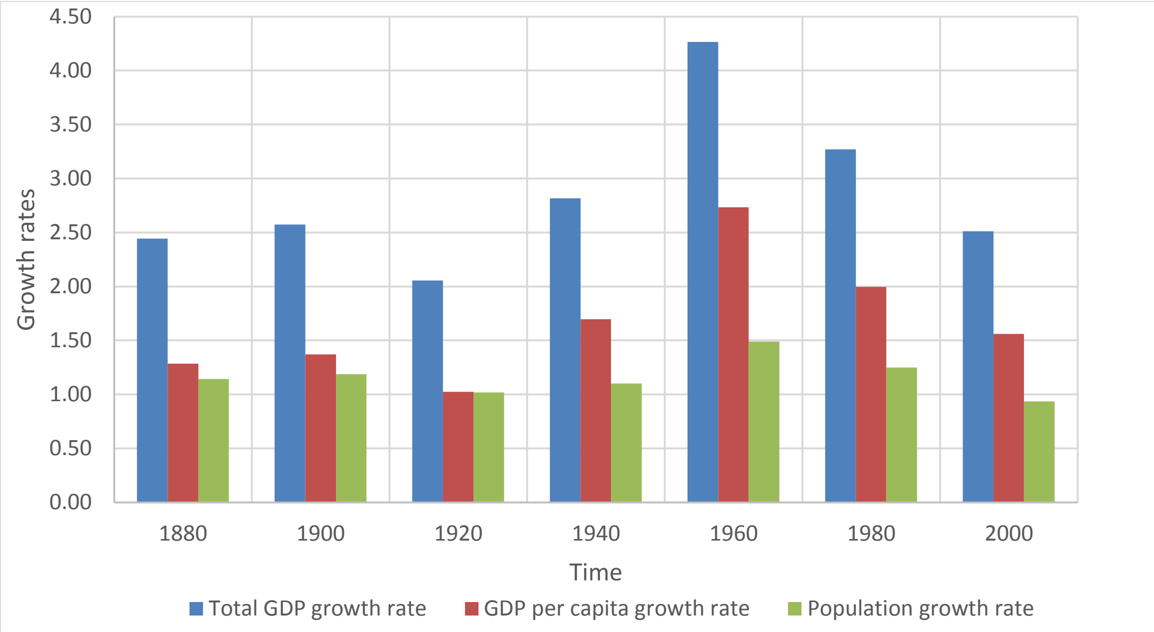
World	Correlation Coefficients
1871-1900	0.33 (0.37,0.28)
1901-1910	0.06 (0.1,0.02)
1911-1930	0.3 (0.43,0.17)
1931-1950	0.21 (0.37,0.05)
Malthusian Époque	**1871-1950** **0.015** **(0.07,-0.04)**
1971-1990	-0.92 (-0.86,-0.97)
1991-2008	-0.89 (-0.83,-0.94)
Modern Era	**1951-2008** **-0.97** **(-0.9,-1)**

Source: *Maddison (2013)*

Note: Figures is bold represent long run correlations between GDP per capita and population growth rate. All other coefficients are short run correlations.

Moreover, we can appreciate the demographic transition effects also on the side of GDP per capita growth and population growth. In fact, given the standard decrease in population growth rate during the demographic transition, we can infer that the share of GDP per capita growth rate in total GDP growth should be increasing itself. In fact, as we can see from Figure 1.6, the share of Total GDP growth devoted to per capita income improvements has strongly increased from the 50's onwards.:

Figure 6: Total GDP growth rate, GDP per capita growth rate and population growth rate for the international sample, 1871-2008



Source: *Maddison (2013)*

In particular, the share of GDP per capita growth rate in total GDP growth has decreased from its initial level of 52% to 49% after the Second World War; the idea that a Modern Regime holds after the 50's is confirmed by the increase in the share of productivity growth in total GDP growth, which reached a level of 62% in the last two decades.

3.2 A Regional Analysis

A complete empirical study of the demographic transition cannot be limited to a World Analysis. With this aim, we dedicate this paragraph to a Continent description of the relation between income and population quantities. What we will find is in line with relevant literature on the relation between per capita GDP and population growth. In particular, we will see that the World has experienced the demographic transition in two different periods of time and with two different groups of countries.

The first group is the one characterizing more developed countries, in particular Western Europe and Western Offshoots (North America and Oceania). These Regions have experienced a very high income growth since the Nineteenth Century as a result of different economic and institutional factors. First of all, this “Early Takeoff” has been caused by a long history of Colonialism (at least for Western Europe); the great availability of raw materials and cheap labour (in some cases supplied by slavery) from Dominions boosted domestic economies on both the supply and the demand side, enhancing output. Second, these countries have been recognized as those with better economic and political institution (concept of inclusive institution, see Acemoglu, Johnson and Robinson, 2005). This process of income expansion has been accompanied by unpreceded technological progress, which has created higher per capita income (expenditure capabilities) and demand for human capital (switched by human capital biased technology); the interaction of the quality-quantity tradeoff and the reduction in child mortality (because of better sanitation and nutrition) produced the optimal environment for a hump-shaped population growth. As described in Chapter 1, the reduction in child mortality is followed, with a short delay, by a reduction in fertility; hence, a short run increase in net fertility was rapidly followed by a sharp decrease in population growth rates.[48]

The second group is the one constituted mainly by developing and underdeveloped countries, namely East Asia, Latin America and Africa. These Regions have suffered exactly the opposite conditions; rural economies still exist and totalitarian governments are relatively common in Asian and African countries.[49] In this case, the reversing of the Malthusian income-population relationship has taken much more time

[48] Note that this particular behaviour is exactly what is shown by population growth rate in Figure 1.4

[49] Many authors (among all Barro, 1999) proved that democracy comes together with growth.

to show up (at least an extra 50 years), and it still seems to be questionable in some Continents (in Africa at least).

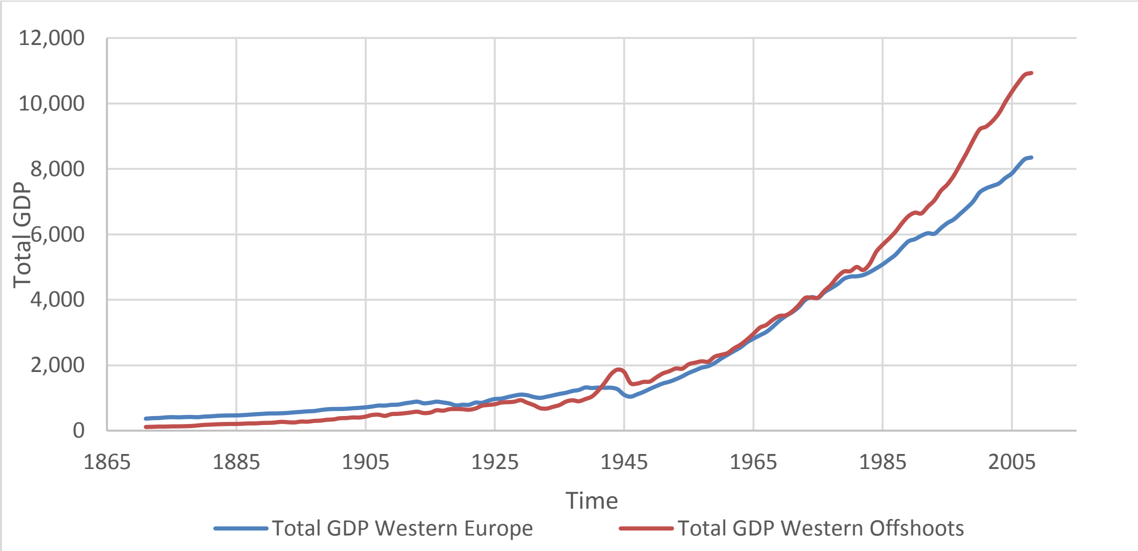
According to Maddison (2004), the World showed the “Great Divergence” in income and standards of living in the first half of the Nineteenth Century (around the 20’s). In the year 1000 the inter-Regional spread was very narrow (about 5:6 in favour of the West); by 2001 all countries had increased their incomes, but the West was 18times richer than the Rest.

The rest of the paragraph develops as follows. In the first part we will analyse income and population variables that constitute the essence of the demographic transition in “The West”; with such an expression, we intend to refer to developed countries, in particular Western Europe and Western Offshoots (North America and Oceania). In the second part we will replicate the same analysis and tests on less developed (or developing) countries, named as “The Rest”; this group is formed by Latin America, East Asia and Africa.

3.2.1 The West

As already introduced, this group of countries is the one that has experienced an earlier and faster economic growth, both in total and per capita terms. In fact, as already stated, it achieved the larger share of World GDP since the early Nineteenth Century, and consolidated its position in the course of decades. We can study total income in the West by plotting Figure 7:

Figure 7: Total GDP, in thousands 1990$, for Western Europe and Western Offshoots in the Period 1870-2008



Source: *Maddison (2013)*

In our sample, we can see that the West held a portion of Total World GDP of 84% from 1870 until 1950, but it rapidly decreased in subsequent years, reaching a level of 76% before the 70's, and then steadily declined until the Twenty-first Century to its actual value (about 72%).

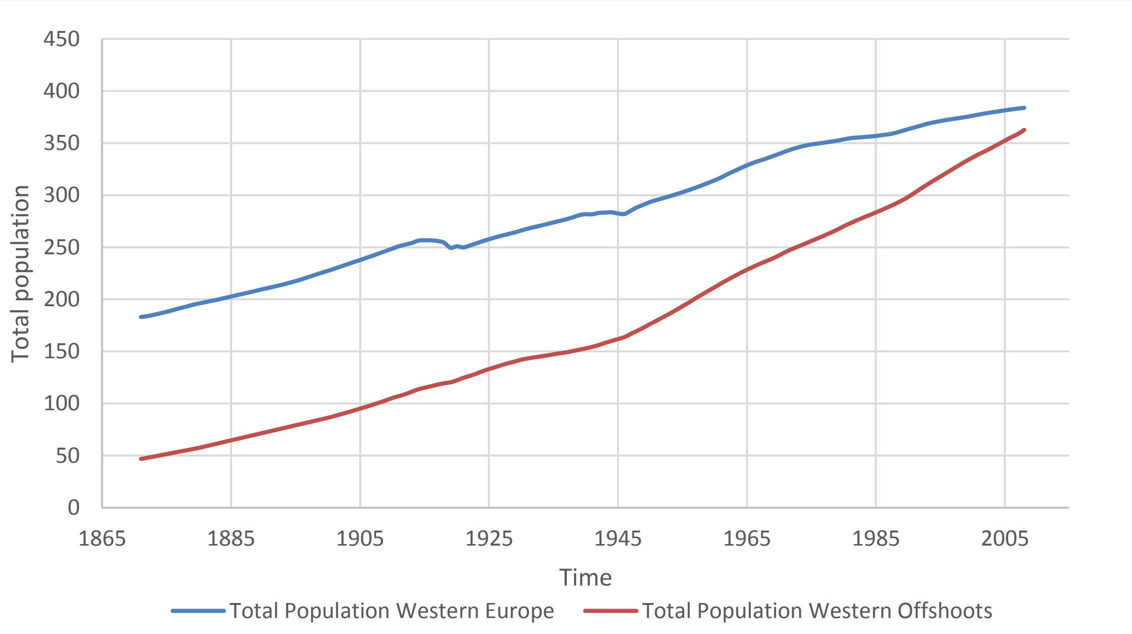
In order to keep such a pace, Western Europe and Western Offshoots experienced an Income Growth that, at least for the Nineteenth and the first half of the Twenties, exceeded the World Growth. In fact, the annual growth rate of output in the period 1871-1910 has always exceeded the 3% (with peaks near 3.4% in the late Nineteenth Century), while the World showed rate of 1 point less. In the interwar period and after the First World War, "The West" started a long period of sustained high growth with rates of 3% and 4% until 1970. After the economic boom, Western Europe and western Offshoots underwent a Sustained Growth path with rates lower than the World' ones, in general from 0.3 to 1 points less.[50]

However the huge increase in total output has not been matched by a equiproportional population growth. Different population growth rate trends can be

[50] In general, authors explain this phenomenon basing on demographic data. A large share of population in working age determined the availability of cheap labour and a light pension and child care system; these society revenues can be reemployed for production purposes. The West has simply experienced what The Rest is facing (or has faced) nowadays.

appreciated in Figure 8:

Figure 8: Total population, in millions, for Western Europe and Western Offshoots in the Period 1871-2008



Source: *Maddison (2013)*

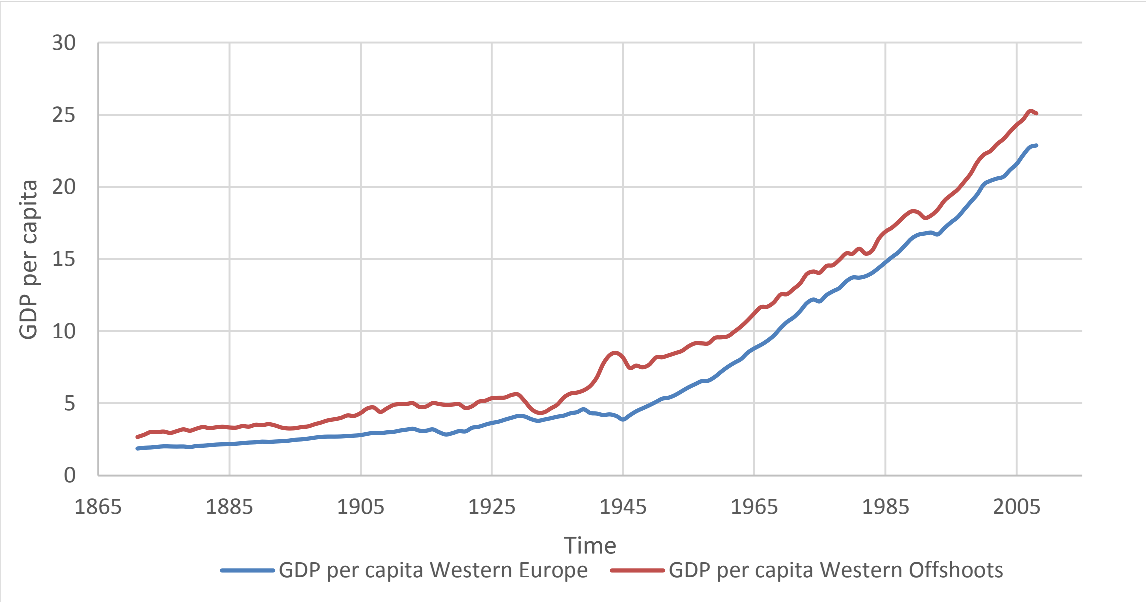
The share of total population in The West is hump-shaped during the last Centuries. In fact, this proportion has increased from the 59% in the 70's of the Nineteenth Century to the 60% around the 1910 and maintained for the interwar period. However, after this point, it started decreasing to 54% in 1950 up to its actual level of 45%.[51] An analysis on population growth rates validates this reasoning. In fact, while World population and Western regions grew at compatible rates until 1930; in the World War times more than double the one experienced by the West (about 1% against 0.4%, for the World and The West respectively), and this kept until nowadays (1.3% for the World against 0.45%).

A peculiar aspect that characterizes these two Regions, the Western Europe and the Western Offshoots, is the geographical or cultural proximity to the United Kingdom; this factor seems to be of primary importance for what has been called "The Takeoff". In fact, long and stable business with England has resulted to be the key element for the transmission of inventories and skills employed in production in the second half of the

[51] These shares are calculated without considering Africa for consistency purposes.

Nineteenth Century; owing to this, income per capita has been affected strongly from the very beginning of the time of our analysis. In fact, we can see that per capita GDP has grown considerably in the considered time range just plotting Figure 9:

Figure 9: GDP per capita, in thousands 1990$, for Western Europe and Western Offshoots in the Period 1871-2008



Source: *Maddison (2013)*

Clearly, as Western Europe and Western Offshoots experienced a higher Total GDP associated with higher population growth rates (in the period 1971-1910), high GDP growth and low population increase (in the War time), and a situation of low GDP and population growth, no relation for GDP per capita growth rate can be stated a priori. However, analysing growth rates associated with Figure 1.8 and comparing them with results of the World Analysis, we can infer some useful information. In fact, we can see that GDP per capita grows much faster in Western Europe and Western Offshoots until the First World War (about 1.4% against 1.3%), then the gap shrinked because of the 1929 financial Crisis (1.1% versus 1%). During the Second World War, the gap grew again (2.96% for Western Regions against 2.7%),[52] and continued until nowadays (about 0.2% of difference in favour of The West). After the 70's, the huge development in East Asia kept the pace with Europe and North America, so that rates of increase are

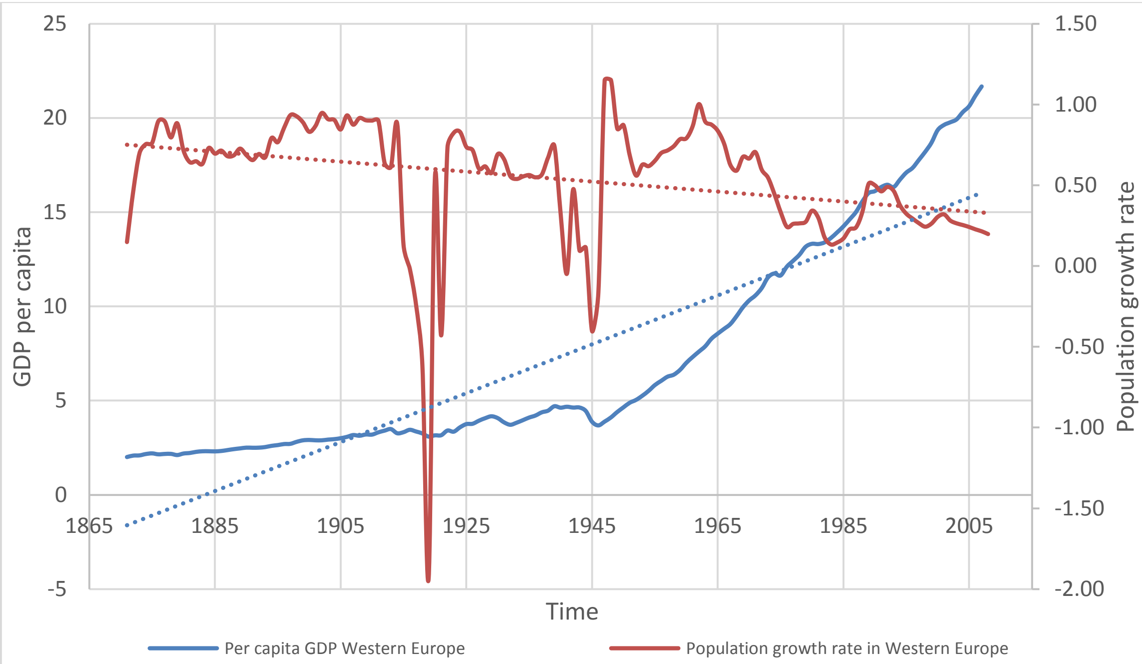
[52] As one can see from the plot, this difference has to be attributed entirely to war expenditures by the US, which constituted an important share of national income.

nearly equal with some regions of the Rest.

Regarding demographic transition, the sharp increase in income and population in the late Nineteenth century should not be disturbing; it could that the rate of growth of population is high, but still it is decreasing. In fact this is what happened, but with different diming for Western Europe and Western Offshoots.

Focusing on Western Europe, we can plot GDP per capita and population growth rate in Figure 10, in such a way to understand whether and when this Region has experienced the demographic transition:

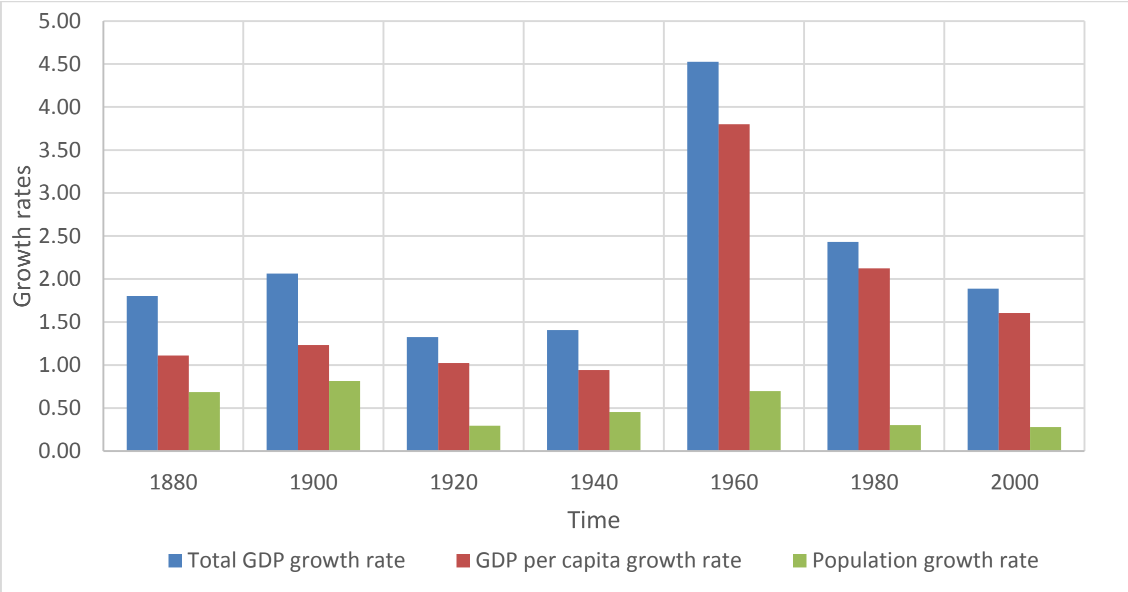
Figure 10: GDP per capita, in thousands 1990$, and population growth for Western Europe in the Period 1871-2008



Source: *Maddison (2013)*

As we can see from Figure 6, there is no relevant period of time in which GDP per capita and population growth rate are positively correlated. In particular, over the entire time range, the correlation coefficient is -0.32; this result validates the idea that Western Europe experienced the demographic transition before 1870 and it income-population relation can be characterized by the Modern Regime definition. This phenomenon can be detected from the increasing ratio of GDP per capita growth rate into GDP growth rate, as shown in Figure 11:

Figure 11: Total GDP growth rate, GDP per capita growth rate and population growth rate for Western Europe, 1871-2008

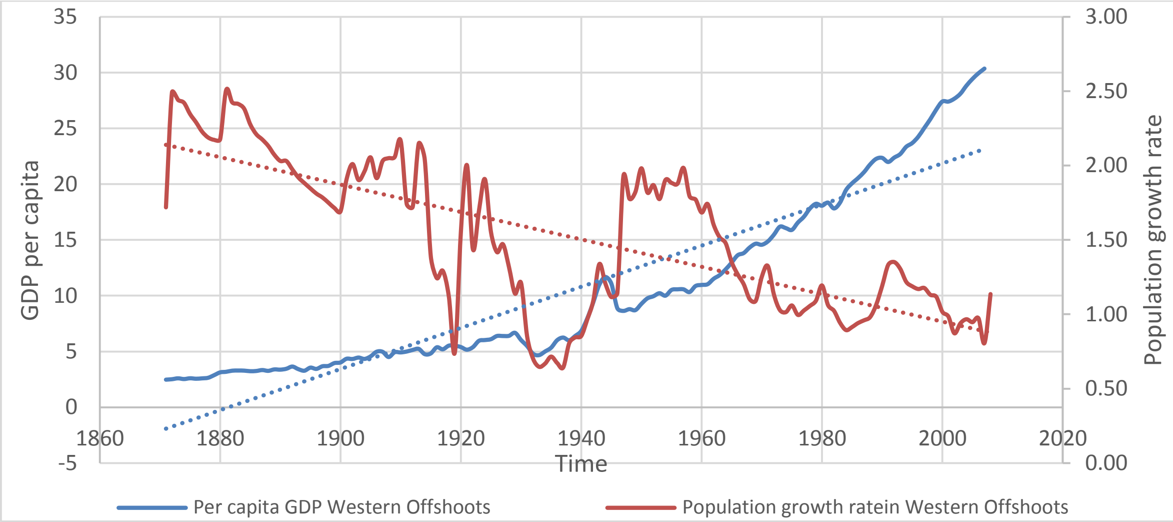


Source: *Maddison (2013)*

As we can see from Figure 11, GDP per capita growth rate has absorbed an increasing ratio of total GDP growth. In fact, this ratio has increased from 60% until the First World War to 78% in the interwar period, to a long run level of 84% till nowadays.

The same situation emerges for Western Offshoots. As it can be seen from the relation between per capita GDP and population growth rate from Figure 12, there is no relevant period characterized by a positive correlation between per capita GDP and population growth rate:

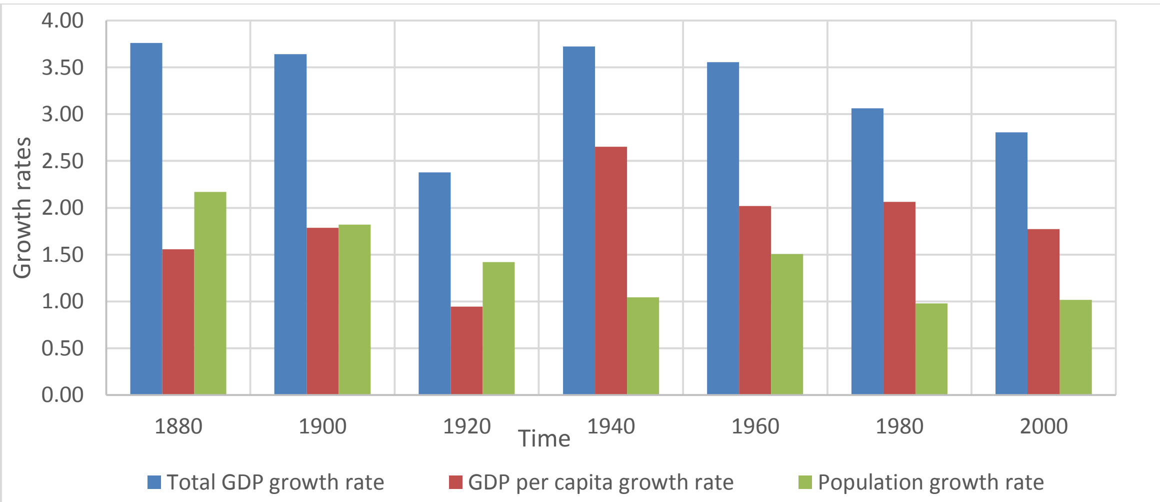
Figure 12: GDP per capita, in thousands 1990$, and population growth for Western Offshoots in the Period 1871-2008



Source: *Maddison (2013)*

As the correlation coefficient between per capita GDP and population growth rate is negative (-0.6) for the entire time range, the demographic transition should have taken place prior 1870 both in Western Europe and Western Offshoots. The same procedure followed for Western Europe applies to Western Offshoots when analysing the ratio of GDP per capita growth into population growth, as in Figure 13:

Figure 13: Total GDP growth rate , GDP per capita growth rate and population growth rate for Western Offshoots, 1871-2008



Source: *Maddison (2013)*

As we can see from Figure 13, the ratio of GDP per capita growth in total GDP growth has increased from 45% before the First World War to 57% after the Second World War, until reaching the long run value of 64%.

As for the international sample, we can summarise correlation coefficients between GDP per capita and population growth rates with 95% confidence intervals even for the West in Table 2:

Table 2: Correlation coefficients between per capita GDP and population growth rate for the West, 1871-2008. Confidence Intervals at 95% in parentheses

The West	Western Europe	Western Offshoots
	1871-2008	**1871-2008**
Modern Regime	**-0.32 (-0.25,-0.4)**	**-0.6 (-0.51,-0.69)**
1871-1900	0.47 (0.52,0.41)	-0.42 (-0.32,-0.52)
1901-1910	-0.05 (-0.028,-0.07)	-0.6 (-0.54,-0.66)
1911-1930	0.47 (0.78,0.13)	-0.34 (-0.17,-0.5)
1931-1950	0.02 (0.2,-0.17)	0.65 (0.86,0.44)
1951-1970	0.18 (0.24,0.13)	-0.97 (-0.83,-1)
1971-1990	-0.33 (-0.26,-0.41)	-0.33 (-0.28,-0.38)
1991-2008	-0.82 (-0.78,-0.87)	-0.87 (-0.79,-0.95)

Source: *Maddison (2013)*

Note: Figures is bold represent long run correlations between GDP per capita and population growth rate. All other coefficients are short run correlations.

4.2.2 The Rest

As for The West, the same analysis can be adopted to study the income population relationship in the Rest. As already introduced, these Regions are those characterized by a much lower level of income (and in some cases even growth of income) per capita form most of the timespan we consider. Different explanations have been given for this huge (now shrinking) divergence have been various. Most of these theories are based on those aspects that have characterized the West; absence of inclusive institutions and abundance of natural resources and labour depleted by Western colonial countries are among the most famous.

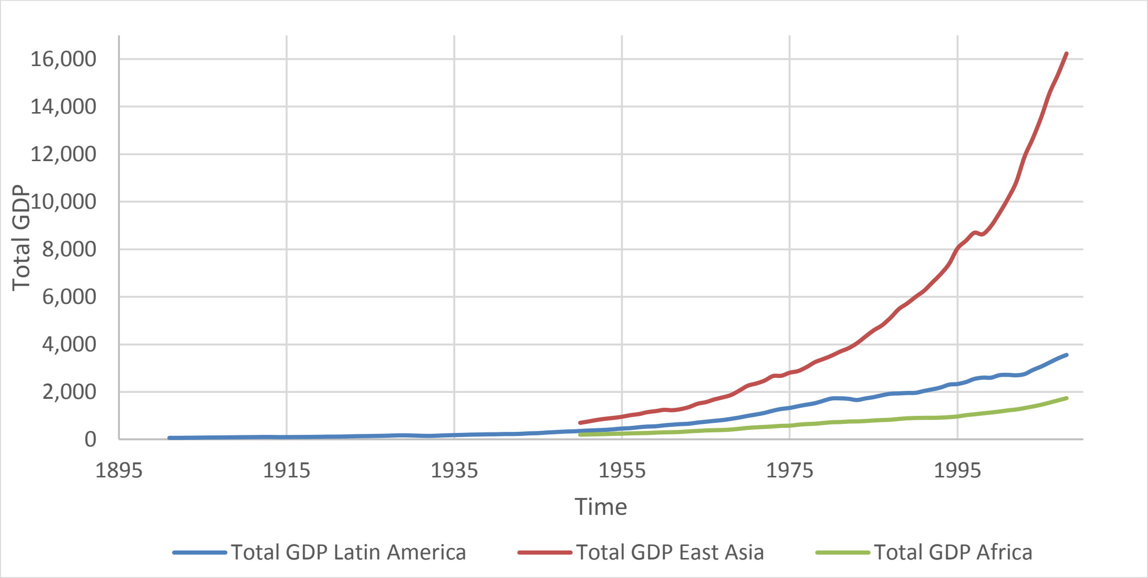
Nevertheless, this explanation seems to fit for Latin American and some African countries only. In fact, even the US, Canada and Australia have been English colonies for a long time, like most of the Asian South East; such a difference, in terms of economic performance, is still a matter of question for scientific literature. [53] Anyway, it is evident that these Regions have greatly recovered from a state of stagnant income level after the West suffered Oil Crises (in the 70's). The great availability of working age people (low wage) has given a strong incentive for investments, so that last period growth rates are among the highest both in Latin America and East Asia; in this sense, Africa is, in most of its countries, still waiting for this event to show up.[54] To start the analysis, we first have to plot Total GDP for Latin America, East Asia and Africa as in Figure 14:[55]

[53] Nevertheless, Asian and Latin America countries rapidly covered what they've lagged behind the 70's; for instance, China Total GDP reached the US in 2014, but still GDP per capita is far from being achieved (according to IMF, it is still at a ratio of 1:5). However, this extraordinary growth is heavily threatened because of the adoption of limiting fertility policies.

[54] In Africa, cultural lignage in favor of numerous families is still very strong. Though some countries (e.g Rwanda) income growth has been roughly comparable to many others in East Asia, this has often been counteracted by sharp increases in population, leaving per capita GDP to its (poverty trap) level.

[55] We just present datas from 1990 fro Latin America and from 1950 for East Asia and Africa in Figure 1.15 for representation purposes. Its representation in annual observation would force to exclude some fundamental countries (like China, India and Argentina) because of availability of data. Political conflicts in Latin America give some strangely interpreted business cycles made of few years. Moreover, the difference in scale between Latin America and East Asia GDP, compared to the one in Africa, does not give full intuition of trends.

Figure 14: Total GDP, in thousands 1990$, for Latin America, East Asia and Africa in the Period 1900-2008

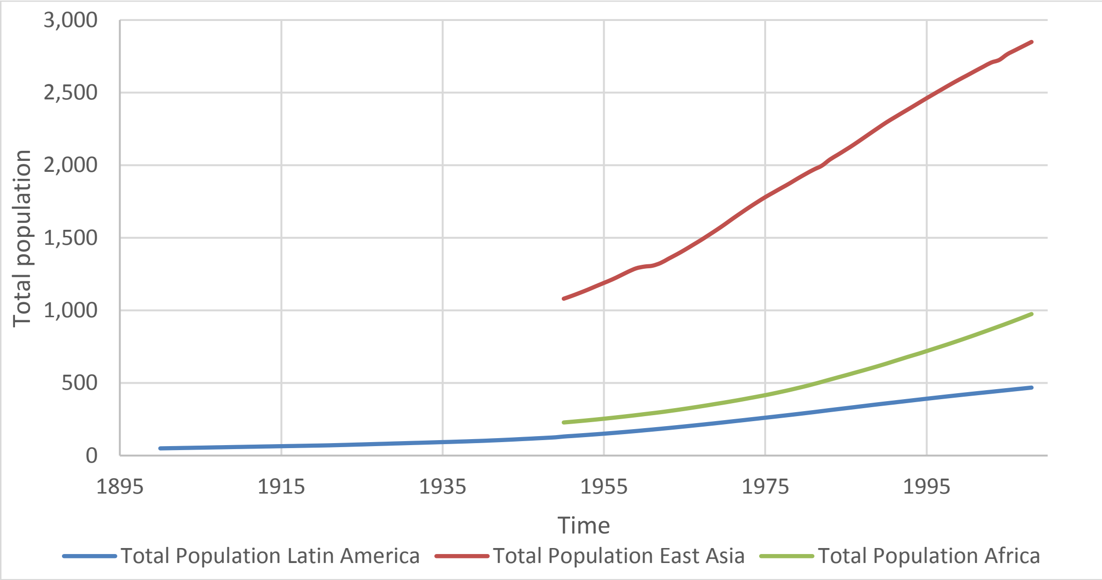


Source: *Maddison (2013)*

From Figure 14, it is possible to calculate associated growth rates. In particular, we can see that total GDP growth rates fit with what already introduced before. In fact, we can see that the growth of total output has been in line with the World rates until 1950, though it differed greatly among Regions; for instance, East Asia managed to reach a total income growth rate that has been double than that in Latin America for about 40 years. However, these Regions succeeded in the so called "Take off" and started increasing at an outstanding pace from the end of the 70's till nowadays. The average GDP growth rate has never been below 3.5% and Latin America gained nearly 4% every year (starting from a growth level of 1.5% just 20 years before). Also Africa, thanks to international investors for oil depletion,

However, a relevant population growth has determined a huge delay in the increase in GDP per capita, as shown in Figure 15:

Figure 15: Total population, in millions, for Latin America, East Asia and Africa in the Period 1900-2008

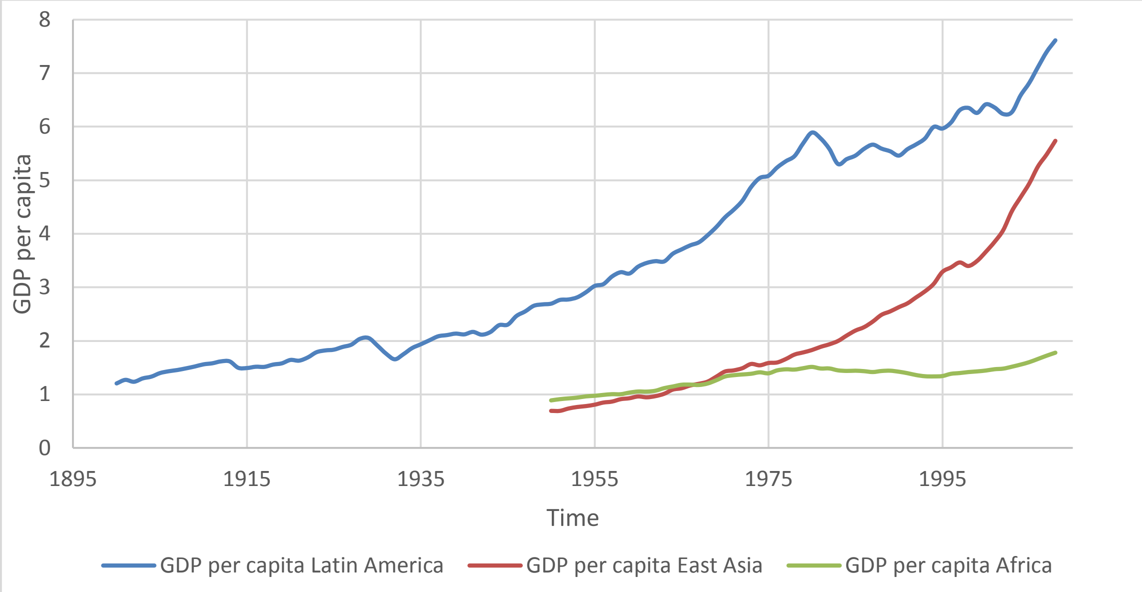


Source: *Maddison (2013)*

In the period we considered, population growth has been roughly stable in all Regions, but it presents the characteristic hump-shaped behaviour of countries during the demographic transition; this peculiarity is clearer in following Figures. In fact, population growth steadily grow from the beginning of the Twentieth (1.8% for Latin America and 0.7% for East Asia), until reaching the peak in the 60's (with 2.8% for Latin America and about 2% for China). Africa has its own story to tell; in these countries, population growth in the 50's and 60's grew at a rate of 2.4%, then it grew to 2.8% and then fall again in the last decade of the century to 2%.

Clearly, income per capita has been influenced by this rapid response of population to income growth, as shown in Figure 16:

Figure 16: Per capita GDP, in thousands 1990$, for Latin America, East Asia and Africa in the Period 1900-2008

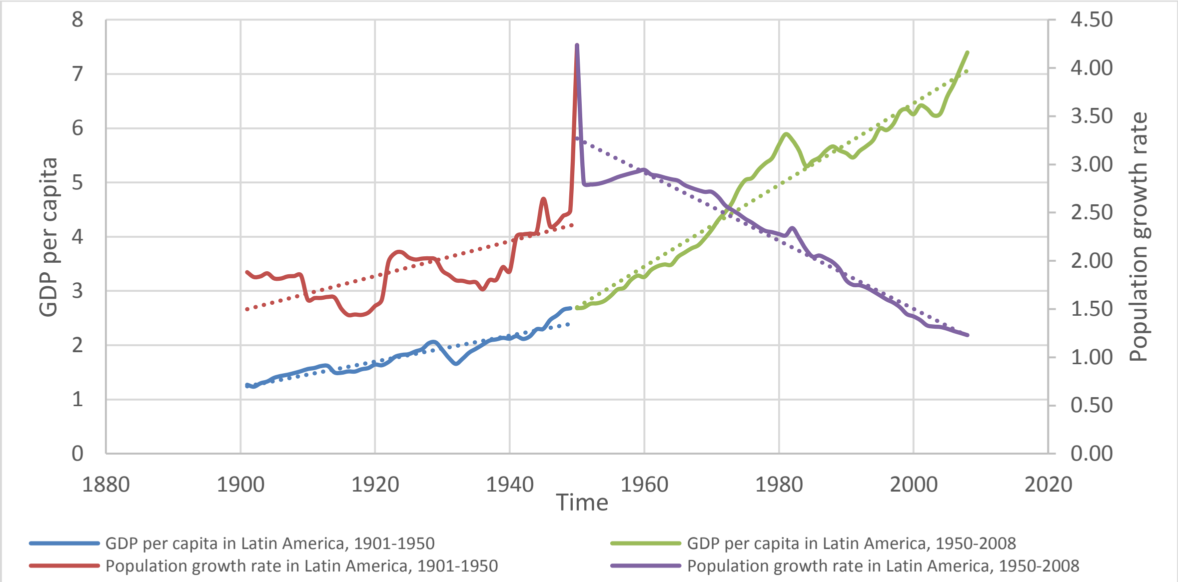


Source: *Maddison (2013)*

Until the Second World War, real GDP per capita grew with rates of about than 1.6% (the West growth rate), ranging from 2.6% to 0.9%. After the War Period, both Latin America and East Asia increased sensitively their growth rates, achieving 2.2% (mostly because of the outstanding increase in growth in Latin America). Finally, the "Take off" and Globalization, did not permit a rapid transfer of GDP growth on per capita capabilities; in fact, GDP per capita did not grow more than in the 50's (about 2.1%). In order for his to happen, the Rest had to wait until the 90's, in which it got to a annual increase of 3.4%, mainly because of the outstanding performance of East Asia (with rates of 4.2%). Other regions lagged behind with rates ranging from 1.7% (Latin America) to 1.3% (Africa) because of lacks in political stability (e.g. financial crack in Argentina in 2001) or retards in those social developments bring to the demographic transition (for Africa, in which GDP growth mainly translated into population growth until 1995).

The same analysis for the West applies for the study of the demographic transition in the Rest; hence we proceed with the comparison of correlation in different periods between income per capita and population growth rates. In this sense, we can figure out the timing of the transition by plotting Figure 16 with growth rates of Figure 15 for Latin America:

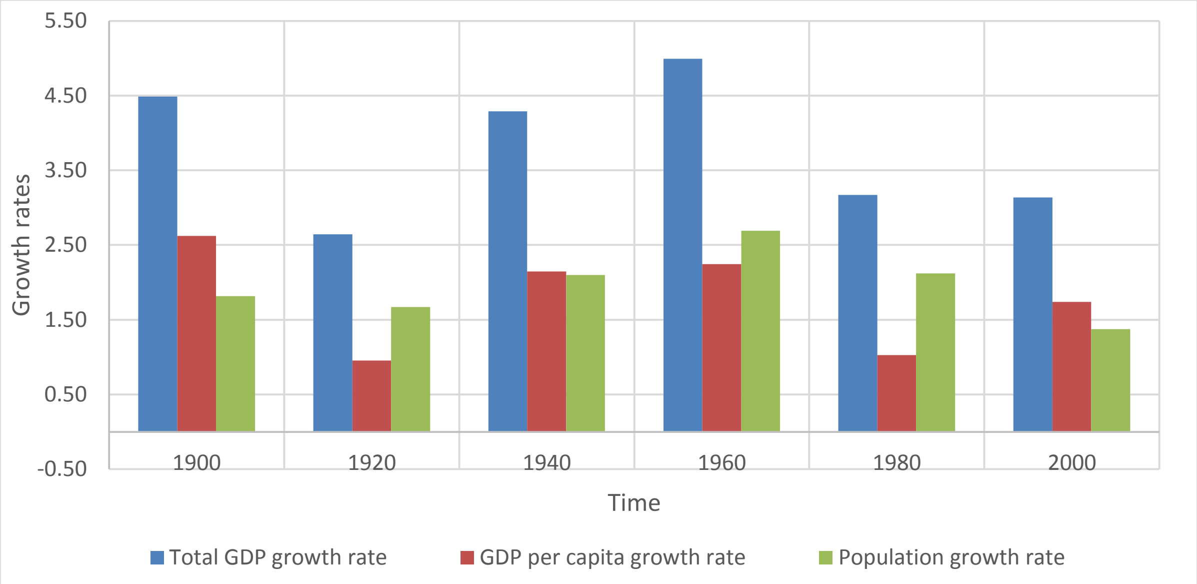
Figure 17: GDP per capita, in thousands 1990$, and population growth for Latin America in the Period 1871-2008



Source: *Maddison (2013)*

From Figure 1.17, it is straightforward to notice that the demographic transition has produced its effects around the early 60’s. This argument is made stronger by many occasions in which GDP per capita and population growth showed a positive dependence between before the 60’s (e.g. the troughs During the First and the starting of the Second World Wars) and a negative afterwards (e.g. the absolutely opposite behaviour in the 80’s). To prove this statement, we can use the following plot in which single series before and after 1960 are investigated in their correlations. Though already evident from the picture, it is possible to see that the correlation coefficient between income per capita and population growth is positive (0.71) before 1960 and negative (-0.93) until 2008. As for the West, we can appreciate demographic transition’s effects on the ratio of total GDP absorbed in GDP per capita growth:

Figure 18: Total GDP growth rate, GDP per capita growth rate and population growth rate for Latin America, 1901-2008.



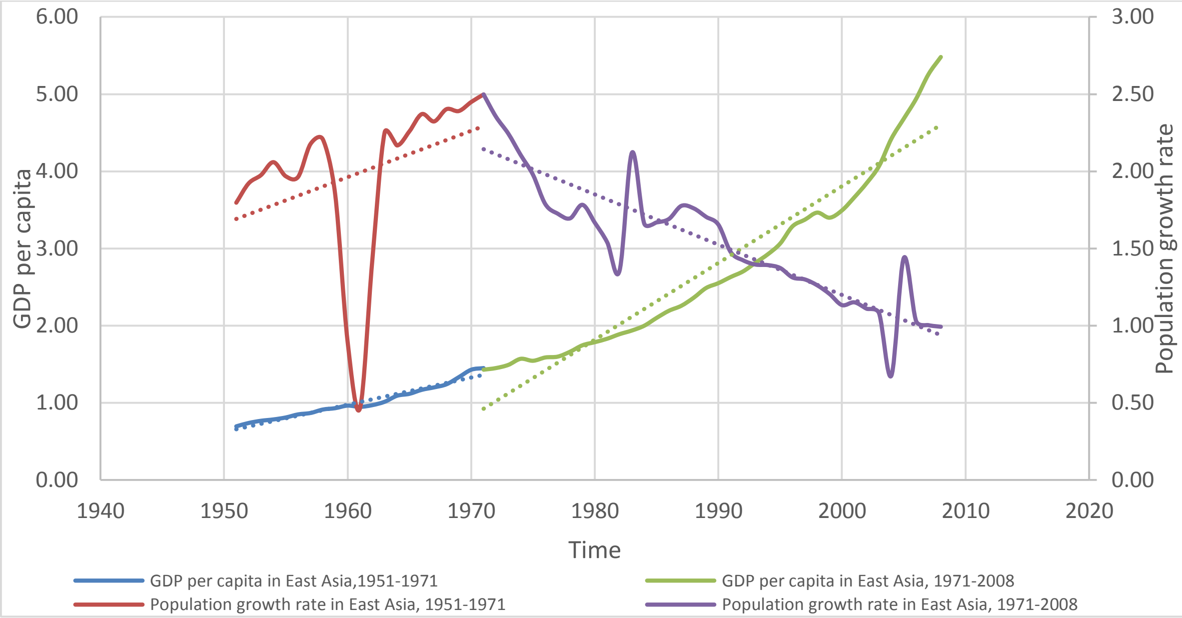
Source: *Maddison (2013)*

In fact, GDP per capita growth share in total GDP growth has decreased from about 58% at the beginning of the last century to 38% in the interwar period; after the Second World War, the share of productivity growth started increasing again up to the actual levelof 50%.[56]

As we have proceeded for Latin America, we will study the East Asia case. In order to study the Asian demographic transition, we plot GDP per capita and population growth rate in Figure 19:

[56] The share of GDP per capita growth in total GDP growth drastically fell to 32% in the early 80's because of the Latin America debt crisis.

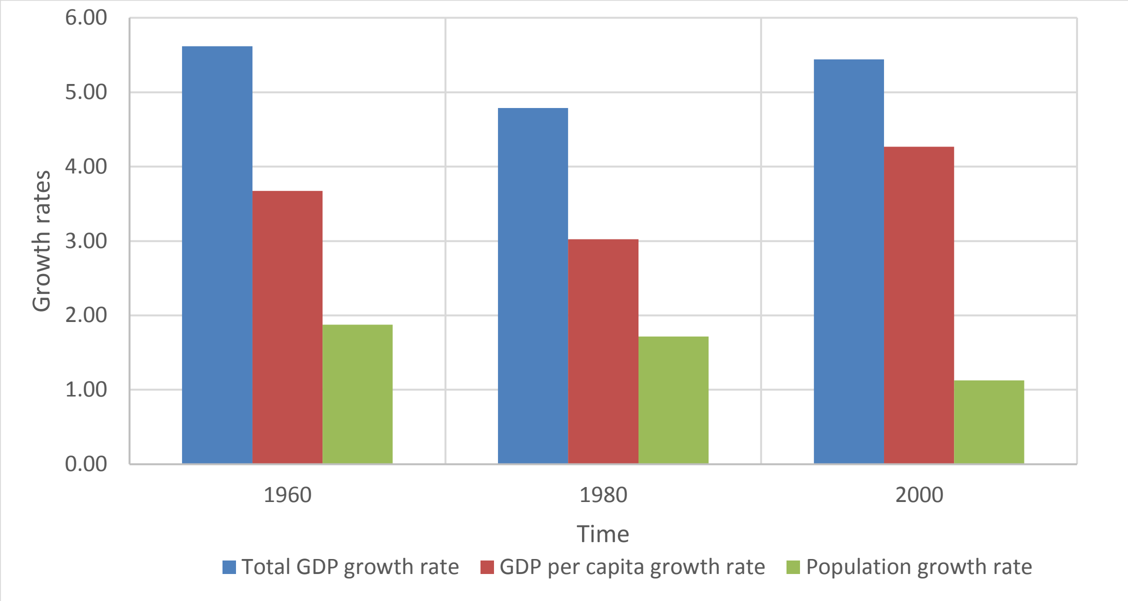
Figure 19: GDP per capita, in thousands 1990$, and population growth for East Asia in the Period 1950-2008



Source: *Maddison (2013)*

We can see that the growth rate of population oscillates during the whole analysis period. However, it is possible to see that two trends are present in the population series, an increasing one until 1970 and a decreasing one afterwards. Moreover, though the collapse at the end of the Second World War (caused by the disastrous defeat of Japan), the income per capita series results to be increasing for the entire timespan. Owing to this, the demographic transition should have come up around the late 70's. In fact, by following the same procedure of Latin America, we can see that the correlation coefficient between income per capita and population growth rate until 1975 is 0.39 and becomes -0.85 until 2008. This conclusion is strongly supported by the huge trough in both GDP per capita and population growth rate experienced at the end of the Second World War.

Figure 20: Total GDP growth rate, GDP per capita growth rate and population growth rate in East Asia, 1951-2008



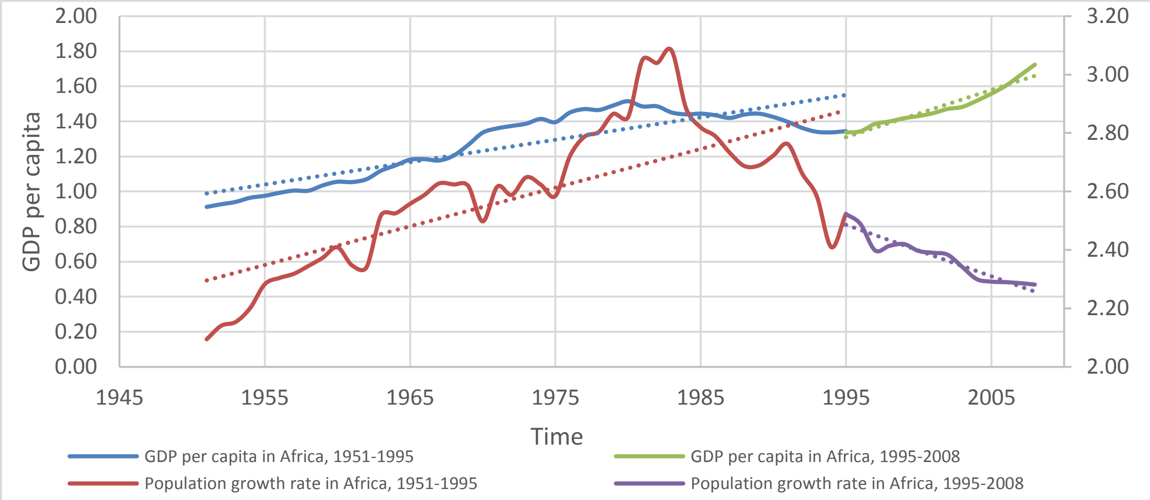
Source: *Maddison (2013)*

According to Figure 20, the share of GDP per capita growth in total GDP growth has decreased in the first two decades (from 65% in 1950-1970 to 63% in 1970-1990),[57] while it started increasing afterwards, reaching a level of 78% in the last twenty years.

Finally, we study the demographic transition in Africa by plotting GDP per capita and population growth rate in Figure 21:

[57] Note that the reduction in the share of GDP per capita growth in total GDP growth is relatively small. This is a consequence of the fact that East Asia experienced the demographic transition in the early 70's, so that half of the decade is characterized by a declining population growth rate.

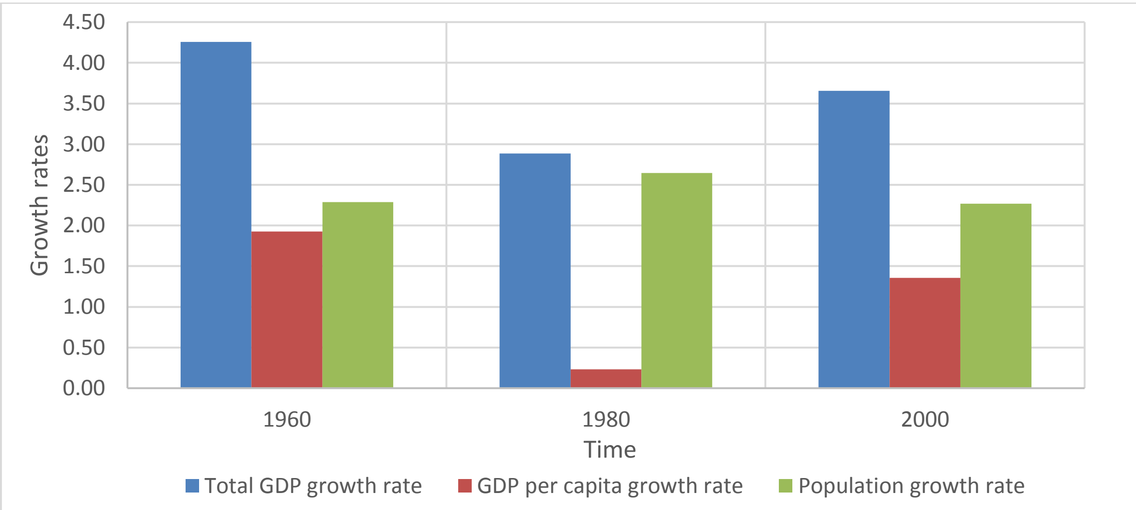
Figure 21 GDP per capita, in thousands 1990$, and population growth for Africa in the Period 1950-2008



Source: *Maddison (2013)*

It is possible to show that series are positively correlated (with a coefficient of 0.9) until 1995, and negatively correlated from 1996 (with a coefficient of -0.9).[58] Also the share of per capita in total income followed this trend, as shown in Figure 22:

Figure 22: Total GDP growth rate, GDP per capita growth rate and population growth rate for Africa, 1951-2008



Source: *Maddison (2013)*

[58] Though population growth rate appears to be decreasing after 1980, also real GDP per capita is stagnant (or possibly decreasing); this is a further proof of the Malthusian framework.

The share of GDP per capita growth in total income growth started from a level of 45% and steadily declined until reaching only the 8%; then, it increased at the beginning of the new millennium up to 37%.

Finally, we plot Table 3 with Correlation Coefficients between GDP per capita and population growth rates for the Rest. Confidence intervals at 95% are in parentheses:

Table 3: Correlation coefficients between per capita GDP and population growth rate for the Rest, 1901-2008. Confidence Intervals at 95% between parentheses

The Rest	Latin America	East Asia	Africa
1901-1910	-0.56 (-0.5,-0.62)		
1911-1930	0.86 (0.98,0.74)		
1931-1950	0.73 (1,0.47)		
Malthusian Époque	**1901-1950 0.72 (0.84,0.59)**		
1951-1970	-0.45 (-0.42,-0.48)	0.39 (0.63,0.16)	0.9 (0.97,0.82)
Malthusian Époque		**1951-1971 0.44 (0.68,0.2)**	
1971-1990	-0.69 (-0.58,-0.79)	-0.58 (-0.41,-0.68)	0.74 (0.81,0.66)
Malthusian Époque			**1951-1995 0.9 (0.97,0.83)**
1991-2008	-0.86 (-0.77,-0.96)	-0.69 (-0.59,-0.8)	-0.72 (-0.65,-0.78)
New Regime	**1951-2008 -0.93 (-0.85,-1)**	**1972-2008 -0.83 (-0.71,-0.96)**	**1996-2008 -0.9 (-0.86,-0.93)**

Source: *Maddison (2013)*

Note: Figures is bold represent long run correlations between GDP per capita and population growth rate. All other coefficients are short run correlations.

Chapter 4: Conclusions

The demographic transition is, no doubt, one of the most important and diffused phenomena that has affected the World as a whole in the last Century. Though some countries have not undergone this phenomenon, the latter is already producing its effects in many Continents. Sharp reduction in mortality rates (in particular for infants) and an enlargement of life expectations at birth shifted resources devoted to rising children in favour of consumption. As a result, population aging and fertility below the replacement threshold caused and an ongoing reversing of the demographic piramid and the unsustainability of a large variety of pension systems (especially the unfunded category, the most diffused nowadays).

In this work we have presented the main theories that have been proposed for explaining the relation between population dynamics and economic growth. Economic Theory has relied for more than one and a half Centuries on the Malthusian Theory; a short increase in income translates immediately into population growth, leaving the economy to a subsistence level of GDP per capita. This mechanism has ruled the World from the beginning of human societies until the late Nineteenth Century, but at a certain point in time something changed. In the Western World, a technological progress boom (Industrial Revolution) gave a strong boost to per capita income, and the relation between output and population has been reversed. The availability of reliable data on demographic and production quantities let this new regularity emerge. The economic answers to this destabilizing phenomenon were among the most articulated ever given to a social issue, because of the outstanding exceptionality and relevancy of the topic for future development. In this paper, three main answer have been developed in a chronological scheme.

In the first chapter of the present work, we describe three main theories that have been proposed to explain the demographic transition.

The first theory moves from the work by Becker (1960) and founds the transition on the heterogeneity of children; offspring are consumption goods whose quality matters for parents. This concept of quality, as already stated, is not a matter of birth, but rather the result of time (opportunity cost of labour wage) and, in some cases, the goods that parents decide to invest in children. This investment is justified by the dependence of parents' utility from children welfare (human capital, future wage or utility, concept of parental altruism), and constitutes the fundamentals for the demographic transition. If elasticity of children quality to income is greater than the elasticity of quantity to income, then economic growth will determine the passage from a state of high fertility to a stage of low fertility (Demeny, 1956). Parents will prefer children quality to children quantity as a result of the maximization process. This result is in contrast with one of the basic assumptions of the Malthusian model, so that fertility is an increasing function of income per capita. Moreover, long run income will result to be higher because of greater human capital accumulation. It is clear that this conclusion leaves space for the debate on schooling systems. In general, economic theory of the quality-quantity tradeoff does not set any fundamental difference between a private and a public education system if heterogeneity of agents is not taken into account. In general, the fundamental role of public provided education is the one of reducing inequality with respect to the situation in which it is not provided (see Doepke, 2004). In fact, the only private schooling would break the society in two categories. One would be characterized by high income level and few better children (New Regime); the other would procreate many unskilled offspring, with few resources escaping a poverty trap (Malthusian World). However, fertility would result to be higher than the optimal one, so that it should be taxed for financing the education sector.

The second theory that has been analysed in the first chapter of the present work finds the cause of the demographic transition in a reduction of infant and child mortality. This theory is based on the concept of surviving children (see Ohara, 1975; Pen Porah, 1976), by which parents are interested in the number of offspring that will survive until a certain period of their lifetime. This assumption changes the Malthusian model in a radical way. In fact, in its original version, mortality rates were a predetermined decreasing function of income per capita. According to this theory, child mortality can be assumed to be exogenous or endogenous. In case it is exogenous, agents perceive they cannot do anything to reduce it, so that they will take it as a parameter when maximizing. In the other case, agents believe that they can reduce it via sanitation (ether private of public) and other forms of child care (human capital endowments, for instance parental control). In the short run, a reduction in child mortality will determine a reduction in fertility as well, in order to keep an equivalence in net fertility; as growth goes by, the activation of a quality-quantity tradeoff will determine a further reduction in fertility and the transition to a New Regime.

Finally, in the third part of the first chapter we present theories explaining the demographic transition as a consequence of the introduction of social security policies. By construction OLG models are suitable for considering heterogeneity of individuals in different stages of their life. In this sense, the most immediate approach would be to consider a childhood period (in which children go to school and accumulate human capital), a working age (in which adults receive a labour wage and maximize utilities with respect to consumption, labour supply and fertility) and a retirement period (in which they consume). Clearly, agents may decide to finance old age consumption in three ways. The first one would be to save income as in models of intertemporal maximization (see Fisher, 1930). The second one would be to invest resources in children, who are going to give material support during the old age; this idea is the so called “old-age security hypothesis”. However, this theory has not been developed in an altruistic framework for obvious reasons. The last possible way to finance old age consumption is by investing resources in a social security scheme, which takes the form of pension systems. In these frameworks, people are taxed on labour income (so the budget constraint during adulthood in negatively affected) and will receive a consumption subsidy when old. The way in which they’ll get back resources in the future gives the distinction between funded and unfunded systems. Anyway, this theory is very interesting for two reasons. First of all, like other theories of demographic transition, it is a real theory of Development; poor rural societies will tend to overinvest in fertility to maintain labour productivity in the future, while developed economies will prefer better children quality. Second, it permits a useful and interesting comparison between the selfish and altruistic hypothesis. In both cases, the dependency of fertility to the introduction of social security systems will be negative (and lower in magnitude in case of altruism), but a change in the interest rate would translate differently according to preferences. If parents are selfish, an increase in the interest rate (so that of the amount of pension transfers) will negatively affect fertility (and positively actual consumption), but the opposite happens if parents are altruistic toward children (see

Cigno, 1992; Cigno and Rosati, 1992), or even if there is mutual altruism.

The second Chapter develops the Barro-Becker (1989) model as an example of intergenerational model with altruism. In this framework, agents live for two periods, but the altruism toward children permits to consider the possibility of infinitively lived agents maximizing instantaneous utilities according to a discount factor. A peculiarity of the model is the concept of pure altruism, by which agents are interested to descendants' utilities and not other measures (for instance child quantity). This altruism function is assumed to be a decreasing function of fertility itself, but still utility is increasing and concave in offspring number; the same happens for consumption. A fundamental and very strong assumption made in the model is the additivity and separability of utility, which stands for time consistency of choices. Though this assumption could result fuzzy in an intergenerational model (the idea that generations will be motivated always and only by altruism, without deviations from a sort of "social contract") it is a logic consequence of homogeneity assumptions. If generations are all equal among themselves, and differ only because of cohort size, this fundamental condition is surely justified. Moreover, the production side is assumed to replicate the Solowian framework. Conclusions that are drawn from this framework are, to some extents, in line with what proposed by approaches studied in the first Chapter of this work. Consumption is a positive function of the net cost of producing another descendant in the same generation. This result derives from the fact that, this in specification, the rate of growth across generations of consumption per person is essentially independent of the level of interest rates, and also does not depend on pure altruism or time preference. On the other side, fertility rises with parental altruism coefficient and the interest rate (same conclusion as in the case of an economy with social securities). This result is derived from the consideration that there is an effect implied by the interest rate that tends to increase consumption over time and this effect dominates the increase in the cost for capital in the steady state. A rise in the initial level of wealth will only have an effect in the short run, with an increase in both consumption and fertility, but it will disappear in the long run. Given that wealth is a component of income per capita (as it can be reinvested in capital accumulation and labour force through fertility), this hump shaped behavior of fertility can be interpreted as the model answer to demographic transition. The introduction of child mortality will decrease (surviving) child rearing cost and so that demand for children; this result is in line with previous literature.

However, other results seem to give an answer to some of previous contradicting conclusions. For instance, the introduction of a social security system will activate a substitution mechanism that will tend to reduce fertility in the long run, and this in line with Prinz (1990) and Zhang (1995); this condition will hold depending on the interest rate and fertility. However, the opposite condition will be triggered if the condition on parameters does not hold, and this validates Cigno and Rosati (1992). Parents would not change consumption patterns (as dynastic wealth has not changed) and will generate less children because of an increase in the cost of rising them (consequence of the Ricardian Equivalence).

Finally, in the third chapter of this work, we propose an empirical analysis for detecting the timing of demographic transition. The method we use is based on the correlation analysis between GDP per capita and population growth. According to Becker (1960), the demographic transition appears in the passage from a positive correlation between income per capita and population growth (or fertility for a one sided analysis) to a negative correlation. The sample we've used to compute plots and tests in obtained from Maddison (2013) and consists in a group of 12 countries (from 4 Continents) for the period from 1871 to 2008. The availability of complete data has been the main obstacle to a more comprehensive exercise. However, results are in line with all main works in the field; a long Malthusian Époque governs the relation until 1950, after which a sharp decline in population growth rate brings the system to the Modern Regime.

We also carry out a Regional analysis for investigating the timing of transitions in different Regions of the World. In this sense, we follow the specification offered by Galor (2004) of the West (Western Europe and Western Offshoots) and the Rest (Latin America, East Asia and Africa); clearly, the availability of data constrained our analysis in the time dimension (for instance we study East Asia and Africa from 1950). Nevertheless results are clear and reliable. The West already presents a negative correlation between GDP per capita and population growth rate since the beginning of the analysis. This result is supported by many authors (e.g. Galor, 2004; Maddison,

2005) and is justified by the strong GDP per capita growth that has been experienced by these regions in the middle of the Nineteenth Century. Finally, the Rest deserves a analysis on its own. In fact, these Regions have been characterized by very low initial values of income, by social and political unrest and bad geographical conditions. All these factors, together with cultures and traditions ingrained into the society in favour of large families, made the demographic transition delay quite long. In fact, we can see that the onset of a reduction in population growth rates has started in 1950 for Latin America, in 1970 for East Asia and 1995 for Africa.

Appendix A

The “World”: Weighted sample of 12 countries from 4 Continents:

- Western Europe: France, United Kingdom, Spain
- Western Offshoots: Australia, Canada, United States of America
- Latin America: Brazil, Chile, Uruguay
- East Asia: Indonesia (and Tomor until 1990), Japan, Sri Lanka.

Regional Analysis: selected group of countries by Continent:

- Western Europe: Austria, Belgium, Denmark, Finland, France, Germany, Italy, Netherlands, Norway, Sweden, Switzerland, United Kingdom, Portugal, Spain
- Western Offshoots: Australia, New Zeeland, Canada, United States
- Latin American: Argentina, Brazil, Chile, Colombia, Mexico, Peru, Uruguay, Venezuela
- East Asia: China, India, Indonesia (including Timor until 1999), Japan, Sri Lanka.
- Africa: Algeria, Angola, Benin, Botswana, Burkina Faso, Burundi, Cameroon, Cape Verde, Central African Republic, Chad, Comoro Islands, Congo 'Brazzaville', Côte d'Ivoire, Djibouti, Egypt, Equatorial Guinea, Eritrea and Ethiopia, Gabon, Gambia, Ghana, Guinea, Guinea Bissau, Kenya, Lesotho, Liberia, Libya, Madagascar, Malawi, Mali, Mauritania, Mauritius, Morocco, Mozambique, Namibia, Niger, Nigeria, Rwanda, São Tomé and Principe, Senegal, Seychelles, Sierra Leone, Somalia, South Africa, Sudan, Swaziland, Tanzania, Togo, Tunisia, Uganda, Zaire (Congo Kinshasa), Zambia, Zimbabwe

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